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email: ijemss@futminna.edu.ng

Assessing the Optimal Investment Portfolio of Pension Funds Under Constraints: Evidence from Nigeria

[1]Henry Siji OLUFAWO; [2]Abolaji Lukman AJIJOLA and [3]Olajide Solomon FADUN
[1]Lagos State University of Science and Technology, Ikorodu, Lagos, Nigeria.
[2,3]University of Lagos, Akoka, Lagos, Nigeria.
olufawohenry002@gmail.com

Abstract

In recent times, several challenges have been confronting the successful operation of pension funds. One of these challenges is pension managers' inability to achieve adequate investment returns on employees' contributions. This has left employees wondering whether pension income can replace their current remuneration in retirement. In the light of this, the study examines the optimal investment portfolio for a pension fund in Nigeria, considering indicators such as risk tolerance, return expectations, and liability structure. A mean-variance optimisation framework was employed to allocate assets across various asset classes, including Equities, Bonds, Bank deposits, and Alternative investments, to assess the optimal level of the investment portfolio. The findings indicate a portfolio yield of 5.30% and a portfolio risk of 13.42%, with asset proportions of 60% government bonds, 10% corporate bonds, and 30% stocks. This shows that the constraint is heavily in favour of government bonds, which prevents the fund from optimising its return. Therefore, relaxing it would improve the risk-return trade-off. The study found that a diversified portfolio with a mix of low-risk and high-return assets delivers optimal returns while minimising risk. It is recommended that pension fund managers adopt more diversified and dynamic asset allocation strategies, increase equity exposure within regulatory limits, and include alternative investments to enhance returns. Regulators should periodically review investment constraints for flexibility, while managers strengthen risk management through modern optimisation tools and robust risk identification and control systems.

Keywords: Optimisation, Pension Funds, Replacement rate, Risk, Yield.

1.0 Introduction

Background of the Study

In most developed economies, pensions play a significant role in providing long- and medium-term funds to both the government and corporate sectors. The mechanisms by which these intermediations occur vary significantly. In those economies with strong, robust capital markets, such as the United States of America, the United Kingdom, Japan, and Canada, pension funds acquire, directly or indirectly, long-term securities, thereby stimulating both the primary and secondary capital markets. Also, in some advanced economies dominated by universal banking, pension funds acquire long-term bonds (certificates of deposit), issued by banks, allowing the banks to provide their corporate clients with loans with different maturities. This has not been the case with Nigeria, according to Ameh et al (2017), pension funds had, until recently, played a minor role as providers of long-term funds to productive investors, as most of their resources were directed to finance public debt, while little was left for the acquisition of real assets. The reasons for this seem to be the underdevelopment of Nigeria's macroeconomy and the poor performance of its financial markets, resulting from a highly uncertain financial environment and the short-term behaviour of most financial supply units in the country.

Pension fund assets in Nigeria have experienced a significant increase in recent years. From 348.3 million dollars before the year 2004 to 32 billion dollars in 2022. (Pencom bulletin 2023). Given the current size of pension assets in Nigeria (over 32 billion dollars, 7.97% of GDP), how pension managers allocate pension fund assets for investment would have a significant impact on the growth of the fund, by extension, the welfare of contributors and the (financial and capital) markets at large.

One of the major challenges confronting pension scheme administration in Nigeria is the inability of the scheme to provide benefits that adequately replace employees' pre-retirement income after retirement. The income replacement rate helps determine how much income will be needed to maintain a retirement lifestyle (Ghadwan et al., 2022). Replacing employee income adequately after retirement seems to be the major challenge in achieving the objective of the pension scheme, which is to enable the retiree to enjoy the same standard of living as he enjoyed before retirement. Currently, the benefits provided by pension schemes in Nigeria do not seem adequate to match the standard of living retirees enjoyed before retirement. This is because many retirees live in deplorable conditions, socially and health-wise, after retirement. According to Nwajogu (2007). Many retirees depend solely on family and friends for survival. Increasing health costs in Nigeria have also made it difficult for most retirees to meet their healthcare and other needs (Saka, Sogunro and Fadun, 2025). There is a need for Nigeria's pension administrator and regulators to review existing investment strategies and adopt more effective, robust ones that will grow and expand pension assets for the benefit of retirees and the Nation at large.

Generally, the study is an attempt to examine the optimal portfolio investment strategies of pension funds in Nigeria under the existing investment constraints

More specifically, the study attempts to identify the existing portfolio investment strategies adopted by pension funds in Nigeria, evaluate the impact of current investment constraints on the performance and efficiency of the pension fund portfolio, determine the proportion of assets that will maximse return in pension fund under existing investment constraints and recommend optimal portfolio investment strategies that can improve returns within the regulatory and market constraints in Nigeria.

To adequately address the objectives of this study, the following research questions were formulated. What are the existing portfolio investment strategies adopted by pension funds in Nigeria? How do current investment constraints affect the performance and efficiency of the pension fund portfolio? What proportion of assets will maximise return in a pension fund under existing investment constraints? And what optimal portfolio investment strategies can be developed to improve returns within the prevailing regulatory and market constraints in Nigeria?

The following hypotheses were formulated to address the objectives. The investment Portfolio in Fund I under the existing constraints is optimal.

i. Pension funds in Nigeria do not adopt any specific or structured portfolio investment strategies.
ii. Investment constraints have no significant impact on the performance and efficiency of pension fund portfolios in Nigeria.
iii. There is an optimal asset allocation mix that maximises return for Nigerian pension funds, given existing regulatory constraints and risk tolerance.

The paper is organised as follows: Section 2 presents the literature review and theoretical framework; Section 3 covers data sources and methodological issues; Data presentation, application of the Markowitz mean-variance model, and discussion are in Section 4; and Section 5 presents the conclusion and recommendations.

2.0 Literature Review and Theoretical Framework

This section reviews the existing literature on optimal investment of pension funds and the relationship between risk and returns, with specific emphasis on Fund I in Nigeria. It also attempts to conceptualise the multi-fund structure of retirement savings accounts in Nigeria and to provide insight into theoretical issues in pension fund investment. The theories underlying the study are categorised into five major hypotheses that underscore the relevance of optimisation in Nigeria. Based on the theoretical literature reviewed, a conceptual framework was adopted for this study, along with the gap in the literature that necessitates it.

Conceptual Review

An investment is a sacrifice of current money or other resources for further benefits. According to Epetimehin (2014), investment involves giving up current resources, typically money, in anticipation of future benefits. This deliberate action by individuals or entities entails allocating funds into financial instruments or assets offered by institutions, aiming to generate desired returns within a specified time frame. An entity's investment can take various forms, including equity shares, deposits, or contributions to a provident fund, allowing for diverse resource allocation to achieve financial goals. Whatever form an investment takes, the bottom line is that money is put to use to earn a return. The challenge is to maximise return by achieving the optimal risk/return relationship. A portfolio is optimal if and only if the corresponding risk-return ratio (σ ɤ, A) is maximised. One way to do this is to apply Markowitz's portfolio optimisation model, known as the mean-variance model.

Regulation on Investment of Pension Fund Assets 2019

Pension funds in Nigeria did not attain their prominence in the financial sector until 2014, when certain provisions concerning the investment of pension assets were amended. Previously, the Pension Act had restricted fund managers from investing pension fund assets in portfolios such as government securities, Treasury bills, treasury certificates, and bank deposits, which have low returns and low risk. This made it difficult for pension schemes to effectively replace contributors' pre-retirement income at retirement. With the 2019 regulatory framework on pension fund investment, pension fund managers can now invest their pension assets in long-term assets such as equities, corporate bonds, offshore securities, and other investment funds. To ensure fair returns and the safety of pension assets, the regulatory body, the Pension Commission (PENCOM), issued a regulation in 2019, the "Regulation on Investment of Pension Fund Assets" (RIPFA 2019), applicable to all pension fund administrators. The purpose of this regulation is to provide uniform rules and standards for the investment of pension fund Assets. The regulation mandated that pension fund administrators operate a multi-fund structure for the Retirement Savings Account (RSA) fund. According to the regulation, the RSA fund is classified into six categories of funds that the PFA can operate, namely Fund I, Fund II, Fund III, Fund IV, Fund V, and Fund VI.

Fund 1, in particular, is for contributors who are risk-takers and are ready to have their contributions invested in ordinary shares, participation units of open- and closed-ended investment funds, and Real Estate Investment Trusts. The regulation further stipulated the investment limit and the maximum exposure of this fund to variable income instruments. According to the regulation, the exposure to variable income instruments of this fund should not be more than seventy five percent 75% of the portfolio value of the funds, the exposure to variable income instruments according to this regulation is the total sum of a PFAs investment in ordinary shares, Real Estate Investment Trust, participating units of open/closed ended and hybrid funds. The investment limit in Government Securities, including sovereign bonds, Global Repository Notes (GRNs), Sukuk, mortgage Bonds, and European bonds, is a minimum of 60%. Investment in corporate Debt is limited to 10% of pension fund assets, and in money market instruments to 30% of pension fund assets. Money market instruments include certificates of deposit, Bankers' acceptances, and commercial paper of corporate entities. Subject to 7.5% of pension assets for any Bank rated “AA”, 5% for “A”, and 3% for “BB”. And for stocks, PFAs are to invest no more than 30% of their pension fund assets in ordinary shares, including global Depository receipts (RIPFA 2019).

Theoretical Review

An optimal investment portfolio is grounded in several theoretical frameworks. The Markowitz mean-variance portfolio selection provided a comprehensive framework for understanding the optimal portfolio problem. The mean–variance model is expressed as follows:

Objective function: Min $\sigma^2\left(R_p\right) = \Sigma\Sigma x_i x_j$ cov $\left(R_i - R_j\right)$

$$R._P = \Sigma x_i \ \ R_j \ \ i, j = 1234 \ \ldots . n$$

Constraints $1 = \Sigma x_i$

Where R_p is the combined rate of return, R; is the yield of the ith stock, X_i and X_j are the investment ratios of security i and j, $\sigma^2\left(R_p\right)$ is the combined investment variance, i.e combined total risk. As simple as the Markowitz asset allocation model is, it has its own limitations.

Gurin (2024) identified four limitations of the Markowitz portfolio optimisation model. First, the model relies heavily on historical data to estimate future returns and risks. However, historical data cannot always accurately predict future market trends, especially during periods of economic or political instability. Second, the model assumes that asset returns follow a normal distribution, meaning they are symmetrically distributed around the mean. However, in reality, market returns are not always normally distributed; extreme events, such as market crashes, can occur. In addition, the model is highly sensitive to input parameters such as expected return and covariance matrix. A small change in this parameter can lead to a significant change in the optimal portfolio. Lastly, the Markowitz model did not account for transaction costs, which can be significant for investors with large portfolios. Transaction costs, such as brokerage fees and taxes, can reduce portfolio returns and affect the optimal asset allocation. It also assumes that investors have access to all available assets, which may not be the case in reality. Moreover, it may not provide sufficient diversification in some cases, especially when the number of assets is limited.

The model may not be fully applicable to investors with underlying risks other than investment risk, for example, insurance companies that pay claims; the risk of claims has to be modelled for optimal portfolio selection. Markowitz's asset allocation model provides a useful tool for balancing the risk-return trade-off. The investors are expected to be mindful of the above limitation and develop other model that will take care of them. Xing, Zhang, & Luo, (2026) explore exploratory mean-variance portfolio selection with constant elasticity of variance models in regime-switching markets. Also, Mei, Zhu, and Chen (2023) examine mean-variance portfolio selection under estimation risk and transaction costs. They conclude that accounting for different transaction costs is equivalent to imposing additional constraints on the portfolio weights, thereby providing desired properties such as sparsity and stability in the trading strategy.

Modern portfolio theory provides a practical method for selecting investments in order to maximize their overall returns within an acceptable level of risk. This mathematical framework is used to build a portfolio of investments that maximises the expected return for the collective, given the level of risk. Markowitz (1952), the pioneer of this theory, argued that most investments are either high-risk and high-return or low-risk and low-return, and that investors could achieve their best results by choosing an optimal mix of the two based on an assessment of their individual risk tolerance. A key component of modern portfolio theory is diversification. The theory argues that in any given investment, an investment's risk and return characteristics should not be viewed in isolation but evaluated in terms of how they affect the overall portfolio risk and return. Meaning that an investor can construct a portfolio of multiple assets that will result in greater returns, without a higher level of risk. As an alternative, starting with a desired level of expected return, the investor can construct a portfolio with the lowest possible risk that is capable of producing that return. Based on statistical measures such as variance and correlation, a single investment's performance is less important than its impact on the entire portfolio.

Baviera and Bianchi (2021) present modern developments in portfolio selection, including the mean–variance optimization model by Markowitz (1956. This model is based on the idea that investors are risk-averse and will only accept higher risk if compensated by higher expected returns. The asset allocation models for an insurance portfolio differ from those considered by Markowitz, since an insurer must pay claims. Zhou and Yuen (2023) proposed a stochastic control framework for insurance firms facing background risk from claims. Their study examined the portfolio selection problem and aimed at determining an optimal investment strategy for insurers. The model considers how an insurance firm allocates its wealth between a risk-free bond and a risky stock while meeting its obligation to policyholders when claims arise. The criterion for portfolio selection is the maximisation of the survival probability (or, equivalently, the minimisation of the ruin probability). The survival probability is defined as the likelihood that the firm's surplus remains non-negative over time, implying that the portfolio selection problem is formulated over an infinite horizon.

Cramer-Lumberg model assumes that continuous trading is allowed; no transaction costs or taxes are involved in trading; assets are infinitely divisible. The model also assumes only two assets in the financial market (Risk free bond and a risky asset, e.g stocks). By the model, the risk-free interest rate is assumed to be non-negative.

The price of the risk-free bond follows $dB(t) = r_o\ B(t)dt$

Where B*(t)* is the price of the risk-free bond at time t, and r_o is the risk-free interest rate which is assumed to be constant $(r_o \geq o)$, the price of the stock following the model

$$dP(t) = \mu P(t)dt + \sigma P(t)dW_t$$

Where $P(t)$ is the price of the stock at time t, μ is the expected instantaneous rate of return of the stock $(\mu > o), \sigma$ is the volatility of the stock price $(\sigma > o)$ and $\{W_t t \geq o\}$ is a standard

Brownian motion defined on the complete probability space (Ω, F, P), while $\{F_t\}$ is the P-augmentation of the natural filtration $F_t^w :=$ the sigma field generated by $\{W_s : 0 \le s \le t\}$.
Assuming the risk process follows the Grammar-Lundeberg model:

$$dR(t) = cdt - ds(t),\ R(0) = s$$

Here c is the constant premium rate and $s(t) = \sum_{i=1}^{N} Y_i$

Where N_t is the number of claims, which follows a Poisson process with intensity λ and $Y_i s$ are individu claims sizes. $\{Y_i\}$ is assume to be independently and identically distributed ii d sequence. In risk theory, c is set to equal $(1+\theta)E[s(1)]$, where $\theta > 0$ is the relative security loading.

The surplus process $X(t)$ can be specified as follows: let $A(t)$ be the total amount of money invested in the stock market at time t by the insurer, assuming $A(t)$ is locally bounded and belong to the set of admissible policies, that is $\{A(t), t \ge 0\}$ is a measurable and F_t adapted process with

$$\int_0^T [A(t)]^2 dt < \infty$$

For every $T < \infty$ we can then specify the model of $X(t)$ by

$$d\ X(t) = A(t)\frac{dP(t)}{P(t)} + (X(t) - A(t))\frac{dP(t)}{P(t)} + dB(t)$$

or

$$dx(t) = (\mu - r_0)\ A(t)dt + r_0\ X(t)dt + \sigma A(t)dW_t + cdt - ds(t),\ X(o) = s\ .$$

Empirical Review

Fund managers constantly seek opportunities to expand and grow their portfolios in order to improve returns. However, such expansion typically leads to higher expected returns alongside increased risk exposure. Chaweewanchon and Chaysiri (2022) highlight this fundamental risk–return trade-off in modern portfolio management.A fund manager considering investments in securities and other instruments faces the problem of choosing from a large number of assets. According to Kelvin (2009), the manager's choice depends on the risk-return characteristics of individual assets in the portfolio; the fund manager would attempt to select the most desirable assets and allocate his funds to this group. Kelvin (2009) asserted that one of the problems fund managers might face is deciding which assets to hold and how much to invest in each to optimise returns and minimise risk.

The optimal portfolio selection problem has practical importance. The Markowitz Model is considered one of the earliest models used in portfolio selection. Markowitz (1956). Lars Peter Hansen (2014) further developed the model by applying dynamic stochastic programming to accommodate dynamic conditions. The study obtained the optimal decision for a consumption-investment model. Gupta, P., & Jacka, S. D. (2023). Applied stochastic optimal control in a continuous-time model to derive closed-form solutions for optimal portfolio strategies under specific assumptions about asset returns and investors' preferences. In a similar study, Gerber and Saiu (2000) considered the problem of optimal capital growth and dynamic asset allocation, using one risky asset and one risk-free asset. The study contended that the Merton ratio must equal the risk-neutral Esscher parameter divided by the elasticity of the expected marginal utility of optimal terminal wealth with respect to current wealth. When there are multiple risky assets, Gerber and Shiu's study explores optimal capital growth and dynamic asset allocation strategies involving a combination of one risky and one risk-free asset.

The study argued that the Melton ratio equals the risk-neutral Esscher parameter divided by the elasticity of the expected marginal utility with respect to wealth. The study also proved the "Two Fund Theorem" (mutual fund" theorem). The two-fund theorem is a framework for building a comprehensive portfolio using mutual funds. The two-fund theorem follows the principle of modern portfolio theory. The theory explains the importance of portfolio diversification and shows how including mutual funds can help limit risk. The concept of mean-variance optimisation, presented by Harry Markowitz (1959) (Weighing risk against expected return), forms the basis of the theorem.

Given the mean-variance optimisation of modern portfolio theory, an investor can identify the optimal portfolio allocation by plotting the efficient frontier and selecting the portfolio on the capital market line. The capital market line is a graph that guides investors in choosing their risk tolerance and investing optimally within the designated allocation at each interval. Insurance and pension fund managers invest in both money and stocks, and the high risk of the latter makes robust investment strategies and risk management crucial (Liu & Yang, 2004). Managers, particularly in insurance and pensions, aim to maximise (or minimise) an objective function subject to various constraints. Yan (2022) investigated the optimal portfolio problem using fractional Brownian motion within a stochastic control framework. The study modeled asset dynamics as a Brownian motion with drift and derived optimal strategies through the Hamilton–Jacobi–Bellman equation, obtaining analytical solutions.

According to Kesto and Legesse (2025), fractional Brownian motion models, particularly those incorporating the Hurst exponent, yield superior projections and more consistent hedging outcomes during periods of crisis. Even when forecasting accuracy is comparable to standard Brownian motion, they enhance risk reduction by capturing long-range dependence. Zhang, Liu, and Zhang (2026) examined the optimal allocation of assets and reinsurance in continuous-time contagious financial and insurance markets. The insurer focuses on contagious claims, which are modelled using an improved dynamic contagion process, and found a dependent structure between the financial and insurance markets.

Pan (2025) found that ZYNP Corporation may be overallocated during periods of declining interest rates due to its sensitivity to interest rate fluctuations. Despite the associated risks, ZYNP's (002448) financial assets remain attractive because of their potential to generate returns above initial investment. This study employs the geometric Brownian motion (GBM) model to estimate ZYNP Corporation's expected price relative to an exchange-traded fund (ETF) (516110). Similarly, Boonen et al. (2025) utilized a diffusion risk model to examine the correlation between two Brownian motions in determining optimal dividend, reinsurance, and capital injection strategies for an insurer operating with two interrelated business lines. Their findings indicate that an optimal dividend strategy follows a barrier approach, where the line with the higher dividend threshold assumes lower reinsurance risk as the aggregate reserve level increases. Capital injection is applied selectively to prevent insolvency in either business line.

Optimal portfolio strategies can grow and expand pension fund assets for the benefit of contributors and the development of the capital market. For example, Cao, (2018) while studying the optimal investment portfolio of enterprise annuity in China submitted that corporate bonds are better than government bonds, also the combination of two or more investment instruments can diversity risks efficiently, Cao, (2018) further asserted that stock investment needed to be selected in the portfolio to earn more return i.e the higher the proportion of stocks investments the greater the yield of the portfolio.

Pension funds are largely influenced by the government through taxes, investment policies, and regulations. Some scholars have studied the effect of taxation on optimal portfolio rules for pension funds, thereby bridging the gap between public and financial economics. In practice, pension fund taxation can be applied at three levels: contributions, gains, and pensions. In Nigeria, taxation applies to pension contributions, while investment gains on those contributions are tax-exempt. Romaniuk (2007), using a pension fund asset-allocation model, asserted that when taxing investment gains or pension income, the participant's optimal policy will become riskier as the speculative fund's absolute value increases.

The taxation-induced distortions are far less pronounced when contributions are taxed, as the only impact on optimal portfolio rules concerns the contribution-hedged fund. Romaniuk (2007) concluded that the current practice of taxing only pensions is not appropriate when seeking to reduce the taxation system's distortions on investors' portfolio behaviour. (Koen and Kees 2008) examined the revenue effect of a cash-flow tax regime for pension savings, using a comprehensive income tax system as a bench mark and Netherland Tax system as a typical case study of a country with funded pension, submitted that taxation of pension implies a major tax revenue loss resulting to an estimated fiscal subsidy of 1.4 to 1.5 percent of GDP.

Many empirical studies on portfolio performance have been conducted; most are based on foreign research (Zheng, 2016; Chaweewanchon & Chaysiri, 2022; Gurin, 2024). Also, a reasonable number of studies have been carried out in Nigeria on pension funds; most of these studies focused primarily on administration and regulation (Fadun, Oyerinde & Isimoya, 2024; Ameh et al, 2017). The few studies on pension fund investment have focused on capital markets and economic development. Some of this work includes that of Oyerinde, Fadun, & Isimoya (2025b); Epetimehin (2012); and Ibiwoye & Adeleke (2008). Studies on portfolio optimisation for pension funds in Nigeria are sparse and have not received adequate attention, to demonstrate how pension funds in Nigeria can grow and expand for the benefit of contributors and the money and capital markets. This study is set to fill this gap.

3.0 Methodology

This section explains the methodology adopted in this study. It consists of sampling method, data collection method, data analysis scope, study scope, and model specification. Purposive sampling was used in this study to select Fund I as the study sample. The justification is that the Fund I investment category is the only fund not subject to pension contributions, i.e., membership of this fund is by application and is open to members willing to take risks. To examine the optimal investment portfolio of a pension fund in Nigeria under investment constraints, using Fund I as our case study, we have considered several variables. These variables are the average annual yield of Government Bonds, Corporate Bonds, Bank Deposits, and Stocks. Data for the study were extracted from the website of the Central Bank of Nigeria and the National Bureau of Statistics.

The study covers the period of twenty years (2004 – 2023). This seems to be the most recent period, and most of the data for the variables considered were not available before it. Hence, we deem it necessary to start our analysis from 2004.

Model Specification

Markowitz has a lot of models for portfolio analysis. For the purposes of this study, the mean–variance analysis model was adopted, with Python software used as an auxiliary tool. The mean-variance model is a follow:

Objective function

Min risk: $\sigma^2\left(R_p\right)=\Sigma\Sigma x_j x_i \quad \text{cov}\left(R_i-R_j\right)$

Max expected return $R_p = \Sigma x_i R_i$

where $\sigma_2\left(R_p\right)$ is the combined investment variance and R_p is the combined rate of return. R_i is the average annual yield of the ith asset, x_i and x_j are the investment ratio of assets i and j and Cov (R_i-R_j) is the covariance between two assets. Covariance measures the direction of the relationship between two variables. A positive covariance means that both variables tend to be high or low at the same time. A negative covariance means that when one variable is high, the other tends to be low. The formula above using Python software can calculate the expected combined rate of return under a certain ratio of each investment instrument, and ensure the risk is minimised. The significance is that investors can predetermine a defined level of return, which can be used to determine the total investment risk of a portfolio plan under different investment ratio.

In line with PENCOM regulatory framework on investment of Pension Fund Assets, the proportion of asset that can be invested in Fund I has been specified as follows: Government securities which includes sovereign bond, Global Depository Notes GDNs, Sukuks, Mortagage bonds: not less than 60% corporate Debt Securities not more than 10%, money market instruments, not more than 30%. Ordinary share, including Global depository receipts, not more than 30% of the pension fund assets.

Method of Data Analysis:

In order to test the optimal level of investment portfolio of pension funds in Nigeria using fun I as our case study, the first step is to determine the average annual yield of each variable so as to determine the proportion of each variable to be invested. The second step is to determine the correlation among the assets to measure their linear relationship. The coefficient of correlation indicates the strength and direction of the relationship between two variables; it ranges between -1 and +1. A zero coefficient indicates no relationship. The final step is to determine the covariance between the assets, which indicates the direction and magnitude of the relationship between two variables and can be positive or negative. A positive covariance indicates that variables tend to increase or decrease together, while a negative covariance indicates the variables move in opposite directions.

4.0 Results and Discussion

Table 1 presents statistical summaries of the assets derived from the data sets using the historical returns from 2004 to 2023 (20 years) for stocks, government bonds, corporate bonds, and bank deposits.

Table I: Descriptive statistics of various assets based on their historical data

YEAR	GOVT BOND x_1	BANK DEPT x_2	CORPORATE BOND x_3	STOCK x_4
Average	1.71	3.34	6.23	8.19
Maximum	16.0	5.01	20.21	54.04
Minimum	-2.62	1.12	-15.14	-36.55
Range	18.62	3.89	35.39	90.59
Std. Dev	3.55	0.84	7.21	23.30
Variance	12.62	0.70	51.94	542.72

Researchers Computation, 2025

Table 1 shows the statistical summaries of the portfolio investment of Fund I. The table showed that government bond (x_1) has the lowest average yield rate of 1.71 percent which is slightly followed by Bank Deposit (x_2) of 3.34 percent, while corporate bond (x_3) and stocks (x_4) have 6.23 and 8.19 percent respecting. In terms of fluctuation, Bank deposits have the lowest fluctuation range of 3.89 with a maximum yield rate of 5.01 per cent and a minimum yield rate of 1.12 per cent, followed by government bonds with a fluctuation range of 18.62 per cent, having a maximum yield rate of 16.0 per cent and a minimum of -2.62 per cent. Making these two investment tools ideal in order to reduce shock, Corporate Bond and Stocks have high yield with average of 6.23 and 8.19 percent respectively; stock having a fluctuation range of about 90.59, maximum yield rate of 54.04 percent and -36.55 minimum yield rate and corporate bonds with fluctuation range of 35.14 having maximum 20.21 percent, and minimum of -15.14 during the period of study.

The standard deviation of Bank deposits is the lowest, at less than 1 per cent, with a modest return of 3.34%, while government bonds have an average yield of 1.71 per cent, with a standard deviation of 3.55 per cent. The standard deviations of corporate bonds and stocks are 7.21 per cent and 23.30 per cent, respectively. These mean stocks are 6.6 times more volatile than government bonds, and corporate bonds are 2 times more volatile than government bonds.

Correlation Analysis

To determine the degree of relationship between the investment assets, a correlation analysis was performed. Correlation measures the linear relationship between the investment assets. The coefficient indicates the strength and direction of the relationship, which ranges between -1 (perfect negative) to +1 (perfect positive) while a coefficient of 0 indicates no correlation.

Table II: Correlation Matrix between assets

	Govt Bond (x_1)	Bank Deposit (x_2)	Corporate Bond (x_3)	Stocks (x_4)
Govt bond (x_1)	1			
Bank Deposit (x_2)	-0.10	1		
Corporate Bond (x_3)	0.10	-0.04	1	
Stocks (x_4)	-0.10	0.09	0.47	1

Researchers Computation, 2025

From the table above, corporate Bonds and stocks have the highest correlation coefficient, which is +0.47. This means that the two investment tools tend to move together moderately. It suggests that the diversification benefits between these two assets are limited. Since corporate bonds and stocks move together in the same direction, allocating to both assets increases the overall risk of the portfolio. Also, government bonds have low or negative correlations with other assets. For example, government bonds have correlation coefficients of -0.10, 0.10, and -0.10 with bank deposits, corporate bonds, and stocks, respectively, making them good for diversification and reducing portfolio risk. From the table, we can see that Bank deposits have weak correlations of -0.10, -0.04, and 0.09 with other assets such as government bonds, corporate bonds, and stocks, respectively, acting as a stabiliser of portfolio risk. Since government bonds and Bank deposits have low negative correlation coefficients of -0.10 and 0.09, respectively, with stocks, diversifying across these assets will help reduce overall portfolio risk. Overexposure to both corporate bonds (x_3) and stocks (x_4) due to their moderate positive correlation (0.47) should be avoided.

Covariance Analysis

Covariance measures how much two or more variables change together. It is a statistical concept that helps explain the relationship (in terms of direction and magnitude, which can be positive or negative) between two variables. A positive covariance indicates that variables tend to increase or decrease together, while a negative covariance indicates that variables move in opposite directions.

Table III: Covariance between various assets

	Govt Bond (x_1)	Bank Dept x_2	Corp Bond x_3	Stock (x_4)
Govt Bond (x_1)	12.62	-0.30	2.45	-8.27
Bank Deposit (x_2)	-0.30	0.70	-0.25	1.83
Corp Bond (x_3)	2.45	-0.25	51.94	78.65
Stocks (x_4)	-8.27	1.83	78.65	542.72

Researchers' computation, 2025

From the table above, government bonds and bank deposits have a covariance of -0.3, which means that as one of the assets increases, the other decreases slightly. This is ideal for an investor who is ready to diversify. To reduce portfolio risk, corporate bonds and government bonds have a positive covariance, meaning both assets tend to rise and fall together. This asset combination is not ideal for risk diversification and increases the overall risk of the portfolio. The table also showed that stocks correlate perfectly with corporate bonds and correlate slightly with bank deposits; therefore, when combined with the rate of return, we can conclude that stock investment combined with one or two of the three types of investment tools, can reduce investment risk and guarantee a higher yield, which is safe and reasonable for any investor.

Mean-variance estimation of Fund 1

Investment constraints parameter settings

According to the pension fund investment regulatory framework of 2019, the proportion of total pension fund net assets that may be invested in various assets, such as bank deposits, government bonds, corporate bonds, and stocks, is as follows. Government bond (x_1) not less than 60%, Bank deposit not less than 30% corporate bond (x_3) 30%. Combined with the means-variance model, the investment constraint is set to $0 \leq x_3 \leq 10\%,\ 0\% \leq x_4 \leq 30\%$. All $x_i \geq 0$ no short selling.

In this study, the constraints for each investment asset were simulated from the lower to the upper bound to determine the optimal asset portfolio proportion under each constraint. For example, a government bond has a constraint of a minimum 60% and a maximum 100% of the net value of the total pension assets for investment. Bank deposits have a minimum 0% and a maximum 30%, corporate bonds have a minimum 0% and a maximum 10%, and finally, stocks have a minimum 0% and a maximum 30%.

The constraints were simulated as follows

Table IV: Combined rate and combined coverable under each investment ratio

	Gov Bonds	Bank Deposit	Corp Bonds	Stocks	Exp. Return	Portfolio Varian ce
Scheme	x_1	x_2	x_3	x_4	R_p	Var
1	94.0%	6.0%	0.0%	0.0%	2.88%	7.62%
2	92.0%	6.0%	0.0%	0.0%	2.96%	7.75%
3	90.0%	7.5%	0.0%	0.0%	3.03%	7.81%
4	88.0%	6.0%	1.0%	1.0%	3.18%	8.10%
5	86.0%	4.5%	1.5%	1.5%	3.35%	8.37%
6	84.0%	3.0%	2.0%	2.0%	3.52%	8.72%
7	82.05	1.5%	2.5%	2.5%	3.69%	9.11%
8	80.0%	0.0%	3.0%	3.0%	3.86%	9.54%
9	78.0%	0.0%	4.0%	4.0%	4.00%	9.85%
10	76.0%	0.0%	5.0%	5.0%	4.14%	10.10%
11	74.0%	0.0%	6.0%	6.0%	4.28%	10.54%
12	72.0%	0.0%	7.0%	7.0%	4.42%	10.91%
13	70.0%	0.0%	8.0%	8.0%	4.56%	11.27%
14	68.0%	0.0%	9.0%	9.0%	4.70%	11.66%
15	66.0%	0.0%	9.5%	9.5%	4.84%	12.04%
16	64.0%	0.0%	9.5%	9.5%	5.00%	12.49%
17	62.0%	0.0%	10.0%	10.0%	5.14%	12.88%
18	61.0%	0.0%	10.0%	10.0%	5.23%	13.15%
19	60.5%	0.0%	10.0%	10.0%	5.27%	13.30%
20	60.0%	0.0%	10.0%	10.0%	5.30%	13.42%

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Determination and analysis of the optimal portfolio

Objective function:

Min $\sigma^2 \left(R_p\right)=$

$$\Sigma\,\Sigma x_i x_i \text{ Cov } \left(R_i - R_j\right)$$

$$\text{Max } R_p = \sum_{i=1}^{4} x_1 \;\; R_j$$

Limitation factors $x_1 \; + \; x_2 \; + \; x_3 \; \times \; 4 \; = 1$

$$60\% \; \leq x_1 \leq 100\%$$

$$0\% \; \leq x_2 \leq 30\%$$

$$0\% \; \leq x_3 \leq 10\%$$

$$0\% \; \leq x_4 \leq 30\%$$

The table V above showed that the higher the expected rate of returns for an investment portfolio, the greater the risk, which conforms with the basic investment law of any portfolio investment. Assuming the expected rate of portfolio return in fund I is 5% minimum under the above constraints, scheme 16-20seems to be feasible based on the requirement of 5% minimum portfolio rate of return, scheme 16 seems to be the best combination that is more consistent with the minimum risk in the mean-variance model, while scheme 20 is best combination in terms of maximum yield.

A cursory view of Table V above shows that as the proportion of stocks (a high-yield and high-risk asset) increases, the portfolio yield and risk also increase. Also, as government bonds, a low-risk, low-yield asset, decrease the portfolio yield, risk increases. Scheme 20 has a portfolio yield of 5.30% and a portfolio risk of 13.42%, with asset proportions of 60% government bonds, 10% corporate bonds, and 30% stocks. It seems the constraint is heavily in favour of government bond, which prevents the fund from optimising its return; therefore, relaxing it would allow better risk-return trade-off. Given this, the fund's investment portfolio under the above constraints does not seem optimal. We therefore reject the first null hypothesis that the investment portfolio in fund I is optimal.

Also, investing in stocks and corporate bonds, an asset of high risk and high return, maximally, under the existing constraints, seems not to optimise portfolio yield in Fund I. This is because, as shown in scheme 20 in the table, where we have a minimum of 60% in government bonds, a maximum of 10% in corporate bonds, and a maximum of 30% in stocks, this constraint, which is heavily in favour of government bonds, has placed a limit on the portfolio yield. This means the portfolio can still achieve a yield above 5.30% in the table if the constraint is relaxed in favour of stocks and corporate bonds. We therefore reject the null hypothesis H:O3, which states that there is an optimal asset allocation mix that maximises returns for the Nigerian pension fund under the existing constraints.

The findings of the study show that pension fund portfolios in Nigeria still conform to the basic law of investment: high risk is rewarded by high returns, as confirmed from the perspectives of both investors and fundraisers. The study also revealed that existing constraints on investment portfolios strongly favoured government bonds at the expense of other assets, such as stocks and corporate bonds, thereby placing a serious limit on the yields of pension portfolios. This shows that the optimal investment portfolio level of pension fund I may not have been achieved under the existing constraints. Therefore, there is need to relax the constraint in favour of other assets like stocks and corporate bonds.

5.0 Conclusion and Recommendations

The study recommends that pension fund managers in Nigeria adopt more diversified and data-driven investment strategies rather than relying heavily on low-risk fixed-income assets. By incorporating a mix of equities and other viable asset classes within regulatory limits, funds can improve returns while maintaining stability. There is also a need for regulators to periodically review investment guidelines to ensure they remain flexible and responsive to changing economic conditions such as inflation and interest rate movements.

Furthermore, pension fund administrators should apply modern portfolio optimization techniques and strengthen risk management practices through tools like scenario analysis and stress testing. Asset allocation should be more dynamic, allowing adjustments in response to market trends rather than remaining static. Overall, a balance between regulatory compliance, strategic diversification, and active portfolio management will help improve the performance and efficiency of pension fund investments in Nigeria. From the empirical analysis, we can see that the higher the proportion of high-yield assets like stocks in the Nigerian pension funds, the higher the expected rate of returns and the higher the risk, which may be related to the immature capital market and the volatility of stock prices in Nigeria.

The result of this study has shown that stock is an important asset that will increase the portfolio yield of pension fund I. This is because as the proportion of stocks increases, the portfolio yield also increases; therefore, fund managers should invest more in stocks to increase the portfolio yield. Given the portfolio's exposure to stock risk due to its volatility, the pension fund manager should invest more in companies with strong stock ratios, while imposing constraints on those with weak stock ratios.

Pension fund managers should put robust risk management policies and procedures in place to identify, measure, and manage risk. The study has also shown that existing investment constraints strongly favour government bonds, which have placed serious limits on the pension fund's portfolio yield. Therefore, the constraint should be relaxed in favour of stocks and other assets, such as corporate bonds, to increase the total portfolio yield. Regulatory bodies and the government should consider restructuring the Nigerian capital market in the long run to reduce stock price volatility and fluctuations to be able to compete effectively with international markets like the United States, which has the most mature capital market in the world.

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