Development of the Concept of Division with Fraction

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Abstract
Concepts are the basic units of thought that underlie human intelligence and communication. The cognitive or intellectual development is a process of the decontextualization of word-meanings by means of child - adult social interactions in which the child's zone of proximal development is fruitfully utilized.In context of various contemporary cognitive theories where interactionism's approach is predominant, it becomes relevant that the meaning and relevance of the words available to the child at any stage are understood critically. From experience, it has been observed that the process of dividing either a whole number or a fraction by another fraction is characterized with memorization rather than understanding the concept of inverting the denominator (Fractional divisor). This paper is designed to develop the basic concept behind the inversion of the fractional denominator through practical exercises that explain the rationale for the inversion of the fractional denominator. The exercises require learners to have understood the concept of addition and multiplication involving fractions along with the knowledge of equivalent fraction. The analysis of the study was carried out using pre- test and post- test and it was discovered that the leaners' confidence, interest, understanding and general attitude towards fraction were greatly enhanced. Learners also identified the rationale for inverting the fractional denominator when dividing either whole numbers or fraction by another fraction. The paper recommends that several practical exercises should be carried out by learners in order to identify the rationale behind the inversion of the fractional denominator. Also all the exercises should be based on familiar activities within the learners' environment.

Introduction:
Constructivism is an epistemology, a theory of knowledge used to explain how we know what we know. This can be of great relevance to the teachers and trainers as referent. This perspective asserts that knowledge resides in the individual and their knowledge is not transferred from the heads of the teachers to those of the learners rather it is the product of the participatory interaction between the learners and the facilitator. Learning and making sense of what happens during any learning process ultimately rest with the individual child. Children's values, beliefs, feelings and perceptions influence their conceptual development.It is therefore, a necessity for teachers (facilitators) to adopt any principle or method that is capable of arousing the learners' values, beliefs, feelings and conceptions in order to assist learners to develop a proper attitude towards learning. It is on this premises that this paper is designed to explain the rationale behind inverting the fractional denominator when dividing a fraction or a whole number by another fraction.Dweck (2002) and her colleagues have shown that students' beliefs about the nature of their intelligence profoundly affects the kinds of goals they pursue and that these goal choices can promote or derail learning.

Education is something more than mere accumulation of knowledge and skills. It must be concerned with developing those skills and attributes that will promote self-discipline, responsibility, self-expression to be able to distinguish between right and wrong, and confidence. There should be an ability to reflect, reason and to analyze. Basic to the development of these skills and attitudes are the ability and the motivation to think critically and communicate effectively with others. The development of a disposition to think and to reflect on experiences provides an impetus towards the more effective use of skills and greater understanding and appreciation of the knowledge and facts being acquired.

National Research Council (2001) identified five independent but intertwined strands in developing proficiency in the learners of mathematics. These includestrategic competence, conceptual understanding, procedural fluency, adaptive reasoning, and productive disposition.Conceptual understanding describes students' grasp of mathematical concepts, operations, and relations. Strategic competence refers to the ability to formulate, represent, and solve mathematical problems. Procedural fluency describes the ability to carry out procedures flexibly, accurately, efficiently, and appropriately. Adaptive reasoning refers to the capacity for logical thought, reflection, explanation, and justification. And, finally, productive disposition reflects a view of mathematics as sensible, useful, and worthwhile, together with a belief in diligence and self-efficacy

There are two fundamental types of concept – every day and scientific concept. The dichotomy between the two types of concept is on the basis of their formation. Everyday concept is formed on the bases of daily contexts and as such could be incorrectly used by children(Vygotsk 1934 & Vygotsky, 1987).Similarly, Schmittau (1993), observed that Vygotsky's everyday concepts as concepts originated from children's daily lives through communication with their family, friends, or community; and thus are closely connected to concrete personal contexts. Children express such concepts through their own words and use them in their thinking without conscious awareness; hence, such concepts are based on subjectivity. For instance, in a series of conversation with a child in primary three in Nigeria, when he was asked to state the meaning of "half", he described that half meant todivide something equally among people. This thought would have been as a result of his everyday concept of sharing biscuits with friends equally without cheating.

Scientific concept on the other hand is conceived on the bases of a system that has developed in human history and therefore lacks concrete contexts. Kozulin (1990) opined that Vygotsky's scientific concepts are based on "formal, logical, and decontextualized structures". Blackwell (1990) compared Vygotsky's scientific concept with mathematical concepts which are based on a system and as such have logic and objectivity. Mathematical concepts are expressed in mathematical language and introduced to children in a formal and highly organized education.

Zack (1999) discussed extensively the relationships between every day and scientific concepts and concluded that the two concepts are mutually dependent in the process of developing children's concepts in daily lives and in school.

Fraction division is often considered one of the most difficult and least understood topics in elementary and middle-school mathematics (Fendel, 1987; Payne, 1976). Children's achievement on tasks related to this topic is usually very low (Carpenter, Lindquist, Brown, Kouba, & Silver, 1988; Hart, 1981).Studies have shown that many teachers hold misconceptions about this topic and need help to provide effective teaching (Ball, 1990; Rule & Hallagan, 2006; Tirosh, 2000).Domoney (2002) and Hannula, (2003) argued that while students may have some facility in using fractions, many of them appear not tohave fully developed an understanding that fractions are numbers. Kerslake (1986)) emphasizes the need for students to understand fractions at least as an extension of the number system. Her report presents some of the difficulties 12 to 14 year-old students have in connection with fractions such as the pupils' inability to recognize that fraction are numbers that can be used in calculations. In order to develop a conceptualknowledge of rational numbers, students should be able to both differentiate andintegrate whole numbers and fractions. It is important to use several models for eachconcept, but two or more related concepts, whenever possible, should be representedtogether so that their relationship becomes clear. The concrete method that can ensure that learners form a clear and unambiguous conceptual knowledge in learning process is through practical exercises.The concept of fractions is an important area of mathematicsthat is applicable to daily life situations and alsoa prerequisite topic to virtually all other mathematical concepts (Percentage, Ratio, Proportion, Variation etc.). In the light of this, the Nigerian basic curriculum makes provision for the teaching of the concept of fraction from primary one.

Fraction has applications in every field of study be it science or arts. It is in recognition of the importance of fraction that makes it necessary for learners (especially children) to form a clear and comprehensive concept that will be useful throughout their life's span. This paper is designed on practical approach to teaching the concept of division of whole numbers or fraction by another fraction which has been identified by a group of researchers in Japan International Cooperation Agency (JICA), (2006) as a serious challenge in the teaching and learning in both primary and secondary schools in Nigeria.It has been observed from the result of this research that if learners understand the basic concept and principles behind any theory theywill be creative, innovative in their thinkingand further expand their understanding of the concept.

When teaching children the concept of division by fractions, it is important to make the process meaningful, practicable and justifiable. This may be done by using a six-step process that helps children to visualize the act of dividing a whole number or fraction by another fraction, understand the process, capable of caring out the division, develop confidence in the division processand finally recognize the need to replace division by fraction with the multiplication of the numerator by the reciprocal of the denominator.

Barge (**2012**) thought of concepts as a complex and true act of thinking that cannot be mastered through memorization. Concept is an act of generalization which lies in the transition from one structure of generalization to another and the process of generalization is completed only at the formation of true concept.

In developing the concept of dividing either a fraction or whole number by a fraction, learners should be thoroughly taken through the basic meaning of two important words – Sharing and Multiple. The clear understanding of the meaning of these words is crucial to the formation of clearer concept of division by fraction. Sharing-conceptdemonstrates how the numerator is shared amongst the denominator while multiple-concept explains how many of the denominator is in the numerator. In most cases, the precision and fluency in the executionof the skills are the requisite vehicles to convey theconceptual understanding.

Student's re-occurring poor performance in mathematics in primary and secondary schools in Nigerian has become an educational cancer. This worries every parent, guardian and educational stakeholder in Nigeria. Adeniji (1998) and Amoo (2001) expressed tales of woes about low achievement in mathematics in Nigeria secondary schools. Formation of proper concepts is an effective activity in the teaching and learning of Mathematics in order to demystify the abstract nature of Mathematics and one of the sure methods for the demystification of abstract concept is through practical exercises that can engage learners to perceive, carry out activities and observe the results of their activities. Experience has shown that teachers rarely use practical procedures in presenting the concept of division by fraction (JICA, 2008)therefore leave learners with the option of memorizing only the techniques of manipulation. This paper is designed for learners to form a proper concept when dividing a fraction or whole number by another fraction and to justify why one has to "invert" the fractional denominator in the division process

Purpose of the Study

The purpose of the study is to develop the concept of dividing a fraction or whole number by another fraction and to justify why one has to "invert" the fractional denominator in the division process

Research Questions

The following research questions were used for the study.

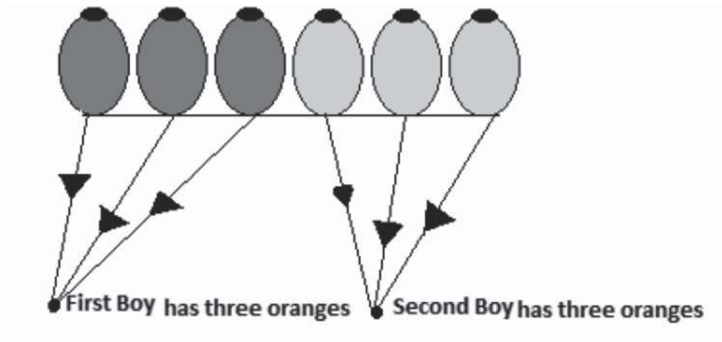
1. How do we divide a whole number or a fraction by another fraction?
2. Why do wehave to invert the fractional denominator when dividing a fraction or a whole number by a fraction?

Methodology

Survey research design was adopted for the work due to the nature of the study. Two groups were identified in each of the schools that have been used for the research. Questions were set to test the learners' skill in division of fractions and some word problems involving fraction. The result of the test shows that 80% of the pupils correctly carried out the multiplication skills but only5% could correctly explain the fractional denominator has to be inverted in the division process. However, when the same group was taken through practical exercises on the process of division by fraction, the learners successfully and confidently carried out division by fraction with explanation. This demonstrates that the concept of the division was not properly formed prior to the conceptual facilitation. The research was conducted for pupils in primary six at an average age of ten (10) years) and NTI study centre in Kwali Area Council of Federal Capital Territory, Abuja.

Seventy-five (**75**) learners were involved in the exercise, thirty-five **(35**) from primary five and forty (**40**) students from NTI study center in Kwali Study center.
The thirty-five (35)leaners were randomly selected from two classes of primary six and the forty (40) NTI students were selected from a class containing sixty (60) students to make up the sample for the research.

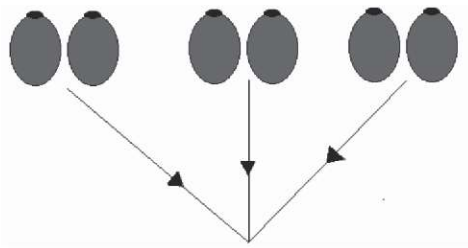
The concept of sharing
Division concept could be studied by designing activities that require the learners to share a given number of familiar objects such as:
If two boys were given six oranges to share amongst themselves, how many oranges will each of the boys get? The learners could orally answer the question but it is recommended that the learners perform the sharing practically i.e.



Therefore $\frac{6}{2}$= 3. This type of exercise should be performed severally with whole numbers so that learners recognize that sharing is the same with division. It is advisable that facilitators should allow learners to perform all the exercises practically and compare the result with their oral answers. These exercises are quite necessary for learners of ages five and six years, there are also very useful when explaining the concept of division to adults who have poor attitude towards Mathematics. However, experience has shown that sharing exercise is NOT quite helpful when the divisor (denominator) is a fraction. It is thereforeappropriate to use the concept of multiple (grouping) in the teaching of division with fraction.

The concept of multiple (Group)
The concept of multiple is similar to sharing concept; however, instead of sharing the numerator by the denominator, learners are to find out how many times that the numerator contains the denominator i.e. how many times that the denominator can be removed from the numerator at once?

Example 1
Divide six oranges amongst two boys ($\frac{6}{2}$), then learners should be asked to determine how many twos are in six (6).

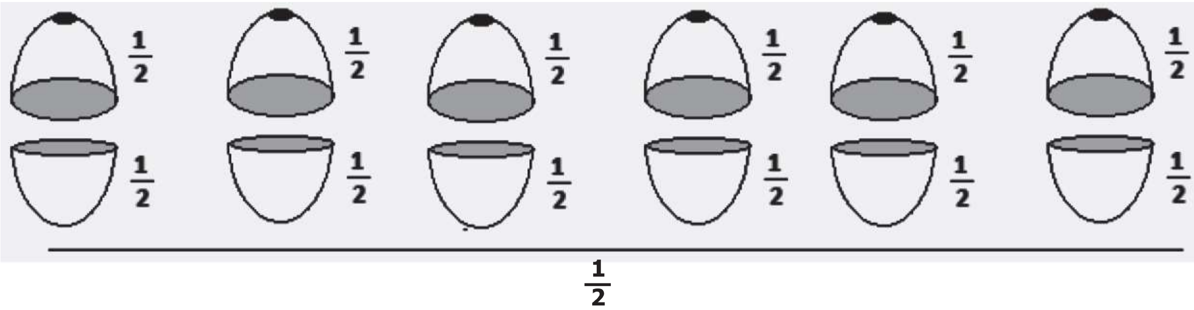


This shows that to take two objects at once from six objects, a learner needs to repeat the process three (3) times. Hence ($\frac{6}{2}$) = 3

The concept of multiple has been found out as the better method to facilitate the concept of division by either whole numbers or fraction. The learners should always ask of number of times the denominator is in the numerator.

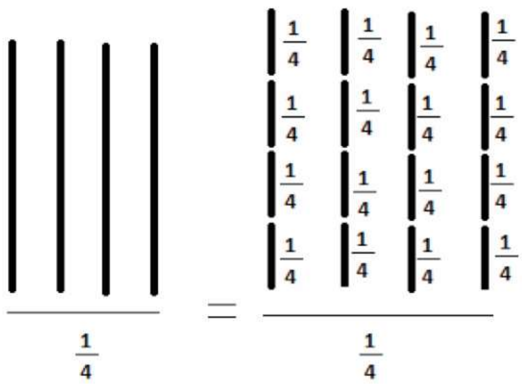
Example 2

Divide six oranges by $\frac{1}{2}$ i.e. $\frac{6}{\frac{1}{2}}$, the concept of multiple requires that learners should ask for how many $\frac{1}{2}$ are in 6 and this can be answered from practical activity as:



It is evident that we have 12 halves $(\frac{1}{2})$ of oranges in six (6) whole oranges. Then the facilitator asks the learners to calculate 6 x. $\frac{2}{1}$ The obvious answer is 12.

Divide four (4) sticks by $\frac{1}{4}$ i.e. $4 \div \frac{1}{4} = \frac{4}{\frac{1}{4}}$, this can be interpreted as four whole sticks contain how many one-quarter stick?This can be answered from practical activity as:



Note: Learners have to count the number of quarters in the numerator. The learners can now count the number of $\frac{1}{4}$ in the numerator. It is evident that we have 16 one – quarter $(\frac{1}{4})$ stick in four (4) whole sticks. Then the facilitator asks the

learners to calculate 4 x$\frac{1}{4}$. The obvious answer is 16. If leaners are taken through such execises severally and compare their results with when the fractional denominator is inverted, they will see the need for the inversion. Experience has showed that learners will form the right concept faster if they start with the whole number as numerators. Even if the numerator is a fraction, similar practical activity will show the need to invert the fractional denominator. If the numerator and the denominator are fractions, there is need to bring the numerator and the denominator to the same denominator using equivalent fraction and learners should at the beginning convert every mixed fraction to improper fraction.

Example 4

Divide $\frac{1}{8}$ by $\frac{1}{4}$ i.e. $\frac{\frac{1}{8}}{\frac{1}{4}}$, then taking the numerator $(\frac{1}{8})$ and the denominator $(\frac{1}{4})$,

But $\frac{1}{4} = \frac{2}{8}$, therefore $\frac{\frac{1}{8}}{\frac{1}{4}} = \frac{\frac{1}{8}}{\frac{2}{8}} = \frac{1 \text{ x } \frac{1}{8}}{2 \text{ x } \frac{1}{8}} = \frac{1}{2}$.

Again $\frac{1}{8} \text{ x } \frac{4}{1} = \frac{1}{2}$. Therefore $\frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{8} \text{ x } \frac{4}{1} = \frac{1}{2}$.

Clearly, the learners would have seen that in all examples there is need to multiply the numerator by the inverse of the fractional denominator.

Example 5

Divide $1\frac{2}{3}$ by $\frac{1}{4}$ i.e. $\frac{1\frac{2}{3}}{\frac{1}{4}} = \frac{\frac{5}{3}}{\frac{1}{4}} = \frac{5 \text{ x } \frac{1}{3}}{\frac{1}{4}}$,

Then taking $\frac{1}{3}$ and $\frac{1}{4}$ to the same denominator, we have that $\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{4} = \frac{3}{12}$.

Therefore $\frac{5 \text{ x } \frac{1}{3}}{\frac{1}{4}} = \frac{5 \text{ x } \frac{4}{12}}{\frac{3}{12}} = \frac{5 \text{ x } 4 \text{ x } \frac{1}{12}}{3 \text{ x } \frac{1}{12}} = \frac{5 \text{ x } 4}{3} = \frac{20}{3} = 6\frac{2}{3}$.

Again, $1\frac{1}{3} \text{ x } \frac{4}{1} = \frac{5}{3} \text{ x } \frac{4}{1} == \frac{20}{3} = 6\frac{2}{3}$

Hence $\frac{1\frac{2}{3}}{\frac{1}{4}} = 1\frac{2}{3} \text{ x } \frac{4}{1} = \frac{5}{3} \text{ x} \frac{4}{1} = 6\frac{2}{3}$

Example 6(**Common denominator approach**)

Divide $\frac{4}{1}$by 2. i.e. $\frac{\frac{1}{4}}{2}$. We know that 2 = $\frac{2}{1}$, therefore $\frac{\frac{1}{4}}{2} = \frac{\frac{1}{4}}{\frac{2}{1}}$, then taking both the numerator $\frac{1}{4}$ and the denominator $\frac{2}{1}$ to the same denominator such as: $\frac{2}{1} = \frac{8}{4}$,

Therefore $\frac{\frac{1}{4}}{\frac{2}{1}} = \frac{\frac{1}{4}}{\frac{8}{4}} = \frac{1 \text{ x} \frac{1}{4}}{8 \text{ x} \frac{1}{4}} = \frac{1}{8}$.

But $\frac{\frac{1}{4}}{\frac{2}{1}} = \frac{1}{4} \text{ x } \frac{1}{2} = \frac{1}{8}$.

Hence $\frac{\frac{1}{4}}{2} = \frac{\frac{1}{4}}{\frac{2}{1}} = \frac{1}{4} \text{ x } \frac{1}{2} = \frac{1}{8}$.

The learners were taken through several practical examples on daily bases for two weeks to consolidate their confidence.

Discussion of the Results

It was observed that after the two weeks of active participation in the activities, all learners who merely memorized the principle of inverting the fractional denominator as a rule discovered the rationale for the inversion and could use it confidently.

Conclusion

Learner's formation of clear and unambiguous concept is the surest way to build their confidence, enhance their positive attitude and interest in mathematics, develop their ability to be creative and encourage them to take career in mathematics related courses. In all the exercises, learners have discovered the reason for inverting fractional denominator in any division process.

Recommendations

In order to motivate the interest and visa-vice the attitudes of teachers and learners towards division with fractions and all Mathematics topics in general, is recommended as follows:

I. Practical demonstrations should be used at the introductory level.
ii. Practical exercises should be carried out severally to ensure proper formation of the right concept.
iii. Examples of all the exercises should be familiar activities within the learners' environment.
iv. The Arithmetic operations (Addition, Subtraction, Multiplication and Division) in fractional problems should be presented to the learners in the order of Addition, Subtraction, Multiplication and Division to ensure the formation of right concept of all the operations.
v. The use of ready-made formulae should be avoided at the introductory stage; leaners should be guided to discover formulae themselves.
vi. Simple and familiar words to the leaners should be used.

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