LINEAR QUADRATIC REGULATOR FOR OPTIMAL ATTITUDE CONTROL OF SMALL SATELLITE

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Abstract

Weight plays a serious challenge in designing the attitude control system for the small satellites. This has reduced the chances of using sophisticated active control mechanism like the actuators, momentum exchange device and gyros for precision control. Gravity gradient stabilisation is suitable for a small satellite because of its light weight, but with a degree of inaccuracy. The accuracy is enhanced when used with active dampers like the magnetic torqrod. Any oscillation in the system is damped within a short time using a state feedback control law to avoid loss of communication between the satellites and their control base station. The concern of this work is to use an optimal Linear Quadratic Regulator (LQR) control technique, to dampen the associated vibration in the gravity gradient stabilised small satellite in the LEO orbit within as short a time as possible, and to show that an autonomous control of small satellite is possible using magnetic torquing only.

Keywords: Small Satellite; Gravity Gradient Stabilisation; Linear Quadratic Regulator; Attitude Determination and Control System; Damper;

Introduction

Small satellites have gained increased popularity since the early eighties, due to the technological advancement in micro-electronics, their relative low cost, and fast turn-around time (from contract to launch). Nevertheless, this comes at the cost of less powerful sensors and actuators, as well as reduced computational power, due to size and weight limitations (Gottfried, 2004). A satellite in orbit is subjected to various kinds of external disturbances classified as either body or traction forces, Kleanthis (2007). These disturbances can alter the satellite orientation with respect to the orbital frame. The Attitude Determination and Control System (ADCS) is an integral part of the satellite operation. It provides continuous information on the relative pointing of the satellite as well as maintains a three-axis stabilised orientation of the satellite, with the bottom part of the satellite facing the nadir direction for continuous communication of the satellite with the ground station. However, the ADCS has traditionally been too complex and expensive for use in small satellites because of heavy weight and complexity. But these satellites must be stabilised in their orbits within a short time when disturbed for effective communication with the base station, hence, an engineering challenge on attitude control system.

Passive stabilisation techniques (gravity gradient method) offered a way of providing some control while staying in compliance with the satellite's power and mass requirements since no energy is required for operation (although some stored energy is required during the deployment of the boom). This system operates in open loop (i.e. no control feedback information is provided from attitude sensors) with low accuracy. However an active control system (magnetorquer) employs actuators to generate a control torque to the satellite which provides position and rate information to close the loop of the control system. By using feedback information to fine tune the control system, the performance values of the gravity gradient attitude control of small satellite will be increased.

Gravity Gradient Stabilisation

The gravity gradient stabilisation of a satellite has been considered as a very attractive method at the beginning of the space age due to its intrinsic simplicity, reliability and low cost of implementation. The main drawback of this passive method of control as explained by Lewis (1989) is its lack of accuracy and the inability to stabilise the satellite oscillations. The structure of a gravity gradient boom is shown in Figure 1.

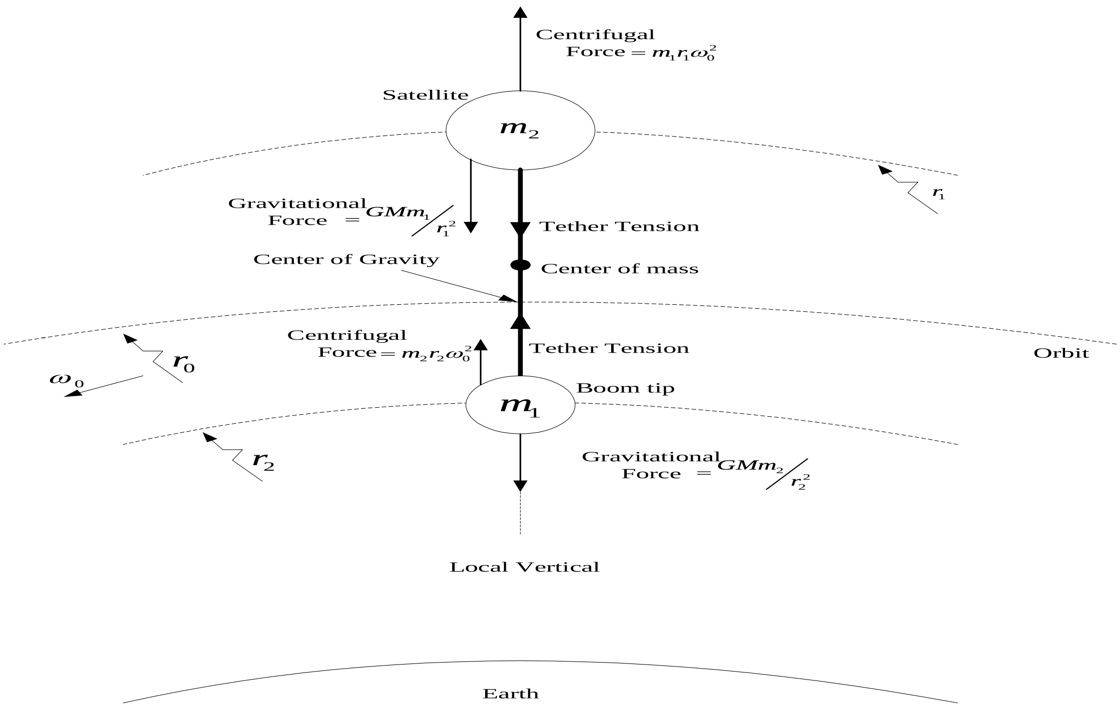

Figure 1: Gravity gradient boom structural arrangement with associated forces

Gravity-gradient stability uses the inertial properties of a satellite to keep it pointed towards the earth. It provides the restoring or stabilising torque but does not damp the oscillation.

Dampers

Dampers are devices used to control oscillation. A common and cheap method used to reduce the undesired oscillations in the gravity gradient stabilisation of a satellite is by the use of passive dampers (Fleeter and Warner, 1989), even though the time to appreciably decrease the oscillatory motion might be very long. For this reason active dampers, like the magnetic torqrods, have been used in the control system of satellite, which interacts with the earth's magnetic field to produce the needed moments to counteract external disturbances to the satellite. Ouhocine *et al.* (2004) compared passive and active dampers for a gravity-gradient stabilised small satellite attitude control methods. They designed a Proportional-Derivative (PD) control algorithm used to damp the satellite oscillations around its equilibrium position. This work uses optimal control technique to design a Linear Quadratic Regulator (LQR) controller for a gravity gradient stabilised satellite.

Geomagnetic Field Model and Magnetic Control

One of the important applications of the geomagnetic field is in the satellite attitude control system. The interaction between the geomagnetic field and the magnetic dipole moment generated within the satellite generates torque that can be used to control the satellite's attitude. This technique has been widely used because it is relatively lightweight; it presents low power consumption and is extremely inexpensive compared to other methods of control (Sidi, 1997). The modelling of geomagnetic field can be in analytical form (with Gaussian coefficients obtained from International Geomagnetic Reference Field (IGRF)) or simplified form (Mohd and Varatharajoo, 2006).

The control torque is influenced by orbit altitude and the residual satellite magnetic field and geomagnetic field. The torque T_m generated is given in equation (1) assuming that both the residual satellite magnetic field and geomagnetic field are orthogonal.

$$T_m = M \times B \qquad \text{...1}$$

where M is the magnetic moments caused by permanent and induced magnetism and B is the geomagnetic field.

The interaction between the magnetic moment M and the magnetic field B, and the generated torque T_m is shown in Figure 2

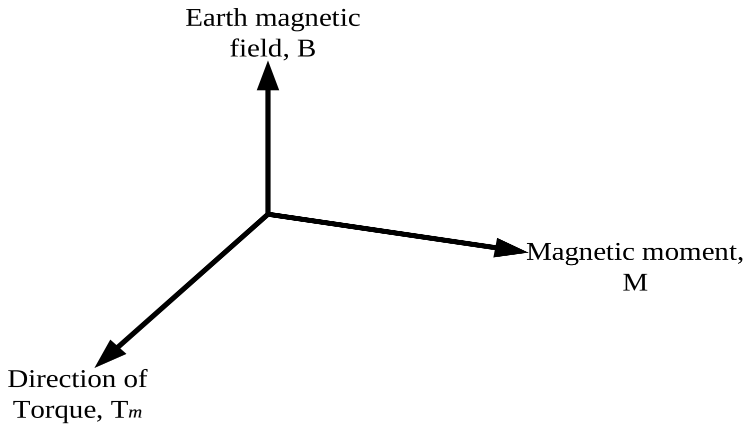

Figure 2: Interaction between the magnetic moment M and the magnetic field B, and the generated torque T_m

Optimal Control and Linear Quadratic Regulator (LQR)

Linear Quadratic Regulator is a method in modern control (optimal control) theory (Kristin *et al*, 2001) that uses state-space model approach to analyse such a dynamic system. These models, which are mathematical representations of the satellite dynamics, are used to study the dynamic response of real systems (Roland, 2001). The LQR control leads to linear control laws that are easy to implement and analyse. The system being controlled is assumed to be at equilibrium and it is desired to maintain the equilibrium despite disturbances. It is termed LQR because the system model is linear and the performance index is quadratic in nature. The word "regulator" refers to the fact that the function of the feedback is to regulate the states to zero (Roland, 2001). This modern control approach is different from the classical control method (Joseph, 2006). It is characterised by:

- Selecting some design matrices that are tied to the desired closed-loop performance
- Introducing an intermediate quantity: the solution to an algebraic equation.
- Solving a matrix differential equation
- Obtaining a guaranteed solution that stabilizes the system
- Obtaining very little insight into the robustness or structure of the closed-loop system.

The LQR-strategy is based on defining a cost function (Joseph, 2006) which should be minimised subject to the system dynamics and then generating a feedback gain matrix for a control law. That is, the optimal control problem is to find a control law u which causes a system:

$$\dot{x} = Ax(t) + Bu(t) \qquad \text{...2}$$

to follow an optimal trajectory $x(t)$ that minimises the performance criterion, or cost function J, in the interval $[t_0, T]$ defined in equation (3) as

$$J = \frac{1}{2}\int_{t_0}^{T} \left(x'Qx + u'Ru \right) dt \qquad \text{...3}$$

Where, $x(t)$ is the state, $u(t)$ is the input and $\dot{x}$ is the rate of change of the state of the system. Also matrix A is an $n \times n$ input square matrix and B is an $n \times m$ control matrix. Both matrices $Q \; and \; R$ are assumed to be positive semi-definite and positive definite respectively, and they are symmetric $(Q \geq 0, \;\; R > 0 \;\; all\, symmetric)$. $Q \; and \; R$ are weighting matrices, or design parameters, where the state-cost matrix, Q, weights the states while the performance index matrix, R, weights the control effort. If Q is increased while R remains constant, the settling time will be reduced as the states approach zero at a faster rate. This means that more importance is being placed on keeping the states small at the expense of increased control effort. If R is very large relative to Q, the control energy is penalised very heavily. The controller is obtained from the solution of a matrix Riccati equation (Roland, 2001):

$$Q + A'P + PA - PBR^{-1}B'P = 0 \quad (as\, T \to \infty) \qquad \text{...4}$$

P is a symmetric, time-varying positive definite matrix and it is the solution to the matrix Riccati equation of equation (4).
Where

$$K = R^{-1}B'P \qquad \text{...5}$$

is the controller gain. And the controller is

$$u = -Kx \qquad \text{...6}$$

taking the Ref as zero (Figure 3).The closed loop system is thus

$$A - B * K \qquad \text{...7}$$

Details on optimal and robust control system design are found in Roland (2001).
The schematic of this type of control system is shown in Figure 3.

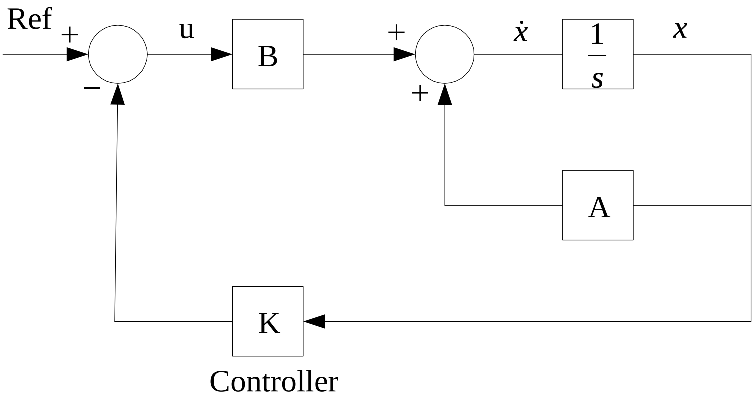

Figure 3: Schematic diagram of system with feedback and gain

Satellite Attitude System Model

The attitude model of the satellite includes both the kinematics and the dynamics of the satellite. The kinematics defines entirely the change in the orientation of the satellite irrespective of the forces acting on the satellite, while the dynamics defines the time dependent parameters as a function of the external forces acting on the satellite.

Attitude Dynamics

The attitude dynamics equation can be analysed using the operator (Sidi, 1997) in equation (8):

$$\dot{A}\Big|_i = \dot{A}\Big|_b + \omega \times A \qquad \text{...8}$$

This states that the rate of change of a vector A as observed in a fixed reference frame (inertial frame) equals the rate of change of the vector as observed in a rotating coordinate system (body frame) with angular velocity ω, plus the vector product $\omega \times A$.

The rotational equations for a rigid body are derived by beginning with the rotational equivalent of:

$$\dot{\bar{h}} = \bar{g} \qquad \text{...9}$$

where $\bar{h}$ is the angular momentum about the mass centre, and $\bar{g}$ is the torque (gravity gradient and magnetic, $T_m + T_{gg}$). This relationship can be represented in matrix form using equation (8) as:

$$\dot{\bar{h}} + \omega_{bi}^{b} \times h = \bar{g} \qquad \text{...10}$$

For a body whose centre of mass coincides with the origin of an orthogonal triad axis frame as shown in Figure 4, where $i, j\ and\ k$ are the respective unit vectors along the body frame axes.

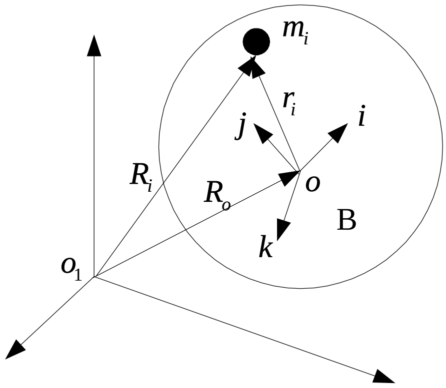

Figure 4: Angular motion of a rigid body (Sidi,1997)

Consider any particle m_i in the body B such that:

$$R_i = R_o + r_i \quad and \quad \dot{R}_i = \dot{R}_o + \dot{r}_i + \omega \times r_i = v_o + v_i + \omega \times r_i \qquad ...11$$

where ω represents the angular velocity vector of the body B with respect to the inertial frame. The moment of momentum of body particle m_i is:

$$\bar{h}_i = r_i \times m_i \dot{R}_i \qquad ...12$$

For a rigid body, $\dot{r}_i = 0$, and the angular motion about the centre of mass of the body $\sum_{m_i} m_i r_i = 0$ holds (Sidi, 1997). Hence,

$$\bar{h} = \sum_{m_i} r_i \times (\omega \times r_i) m_i \quad or \quad \bar{h} = I_m \omega_{bi}^b \qquad ...13$$

Substituting equation (13) in equation (10), we have

$$I_m \dot{\omega}_{bi}^b + \omega_{bi}^b \times I_m \omega_{bi}^b = \bar{g} \qquad ...14$$

And

$$\dot{\omega}_{bi}^b = -I_m^{-1} \omega_{bi}^b \times I_m \omega_{bi}^b + I_m^{-1} \bar{g} \qquad ...15$$

Using the principal axes and writing this in matrix form, we have the nonlinear Euler's equation of motion as:

$$\dot{\omega}_{bi}^b = \begin{bmatrix} \frac{I_{yy} - I_{zz}}{I_{xx}} \omega_2 \omega_3 + \frac{g_1}{I_{xx}} \\ \frac{I_{zz} - I_{xx}}{I_{yy}} \omega_3 \omega_1 + \frac{g_2}{I_{yy}} \\ \frac{I_{xx} - I_{yy}}{I_{zz}} \omega_1 \omega_2 + \frac{g_3}{I_{zz}} \end{bmatrix} \qquad ...16$$

Attitude Kinematics

The kinematics describes the satellite's orientation in space and is derived by the integration of the angular velocity (details in Sidi, 1997). The angular velocity of the satellite model can be described by unit quaternion and skew symmetric matrix:

$$\dot{\bar{q}} = \lim_{\Delta t \to 0} \frac{\bar{q}(t+\Delta t) - \bar{q}(t)}{\Delta t} = \frac{1}{2} S\bar{q} \qquad \text{...17}$$

Where

$$S = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \qquad \text{...18}$$

Equation (17) can be rearranged to give a nonlinear attitude kinematic equation of the satellite as:

$$\dot{\bar{q}} = \frac{1}{2} \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q^{\times} + q_4 I \\ -q^T \end{bmatrix} [\omega] \qquad \text{...19}$$

The system models developed so far are nonlinear (equations (16) and (19)), and such have to be linearised to make the analysis of satellite attitude dynamics easier. This is done within an equilibrium point (Sidi, 1997). The linearised satellite attitude equation for a three-axis stability according to Kristin *et al* (2001) and Wisniewski (1996), for the satellite to always point to the earth with its nadir vector, given in state space form is:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} = A \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} + \frac{-I_m^{-1}}{\|B_m\|} B_m \times B_m \times \tilde{M} \qquad \text{...20}$$

B_m is the geomagnetic field in the satellite body frame and $\|B_m\|$ is its norm. I_m is the moment of inertia of the satellite and $\tilde{M}$ *is the control torque,* $(\tilde{M} = u)$
where

$$A=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -4\omega_c^2 r_1 & 0 & 0 & 0 & 0 & \omega_c-\omega_c r_1 \\ 0 & 3\omega_c^2 r_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_c^2 r_3 & -\omega_c-\omega_c r_3 & 0 & 0 \end{bmatrix} \quad ...21$$

$$r_1=\frac{I_{yy}-I_{zz}}{I_{xx}},\quad r_2=\frac{I_{zz}-I_{xx}}{I_{yy}},\quad r_3=\frac{I_{xx}-I_{yy}}{I_{zz}} \quad ...22$$

$I_{xx}, I_{yy}\ and I_{zz}$, are the moments of inertia about the axes of the body frame and ω_c is the orbital angular velocity.
The input control matrix is:

$$B=I_m^{-1}*G \quad ...23$$

where

$$G=\begin{bmatrix} \left(B_2^2+B_3^2\right)/B^2 & -B_1B_2/B^2 & -B_1B_3/B^2 \\ -B_1B_2/B^2 & \left(B_1^2+B_3^2\right)/B^2 & -B_2B_3/B^2 \\ -B_1B_3/B^2 & -B_2B_3/B^2 & \left(B_1^2+B_2^2\right)/B^2 \end{bmatrix} \quad ...24$$

B_1, B_2 and B_3 are components of the earth magnetic field in the satellite body frame.

B_1, B_2 and

The off diagonal terms of G have an average value of zero while the diagonal terms, defined as $g_x, g_y\ and\ g_z$ respectively have average values that are a function of orbit inclination. The dependence according to Barry (2003) is shown in Table I.

Table I: Average components of Earth Geomagnetic Field against Inclination at 560km: IGRF 2000 (Ref.: Barry, 2003).

Inc. (degree)	g_x	g_y	g_z	Inc. (degree)	g_x	g_y	g_z
0	0.967	0	0.804	*60*	0.739	0.857	0.39
10	0.995	0.068	0.781	*70*	0.709	0.923	0.353
20	0.922	0.256	0.711	*80*	0.691	0.965	0.335
30	0.876	0.46	0.614	*90*	0.686	0.981	0.333
40	0.826	0.632	0.522	*100*	0.691	0.965	0.335
50	0.78	0.762	0.446	*110*	0.709	0.923	0.353

Results and Discussion of Results

The control law was tested by performing simulations with satellite configurations and initial conditions obtained from Ouhocine *et al* (2004) as shown Table II. The control tuning matrices R and Q were obtained through iterative process following expectable requirements such that the system damps to the desired equilibrium within limited time with allowable control effort.

The initial conditions for roll, pitch and yaw, and the weight matrices used in the simulations are shown in Table III and Table IV.

Table II: Satellite characteristics (From: Ouhocine et al (2004)

Moment of inertia I_x	$100\text{kg}/m^2$
Moment of inertia I_y	$100\text{kg}/m^2$
Moment of inertia I_z	$2.5\text{kg}/m^2$
Altitude h	560km
Inclination β	60 degree
Orbital rate ω_c	0.0010764 rad/sec
Desired Euler angles $[\phi, \theta, \varphi]$	$[0,\ 0,\ 0]$rad
Simulation Time	40 seconds

Table III: First initial condition of the satellite attitude and the weight matrices for simulation

Roll, Pitch, Yaw, in rad.	$[0.0524 \quad 0.0175 \ \text{-}0.0524]\, rad. = 3^0, 1^0 \ -3^0$
Q	$Diag.([100\ 100\ 100\ \ 0.01\ \ 0.01\ \ 0.01])$
R	$I_{3\times3}$

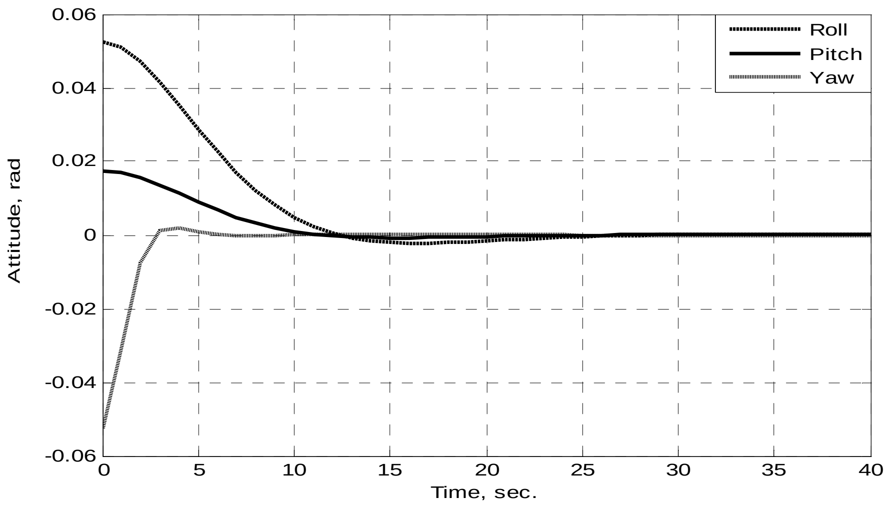

Figure 5: Simulation responses for LQR controller

Table IV: Second initial condition of the satellite attitude and the weight matrices for simulation

Roll, Pitch, Yaw, in rad.	$\left[1.3963 \quad 1.0472 \quad -1.3963\right] rad. = 80^0,\ 60^0\ -80^0$
Q	$Diag.\left(\left[100\ 100\ 100\ \ 0.01\ \ 0.01\ \ 0.01\right]\right)$
R	$I_{3\times3}$

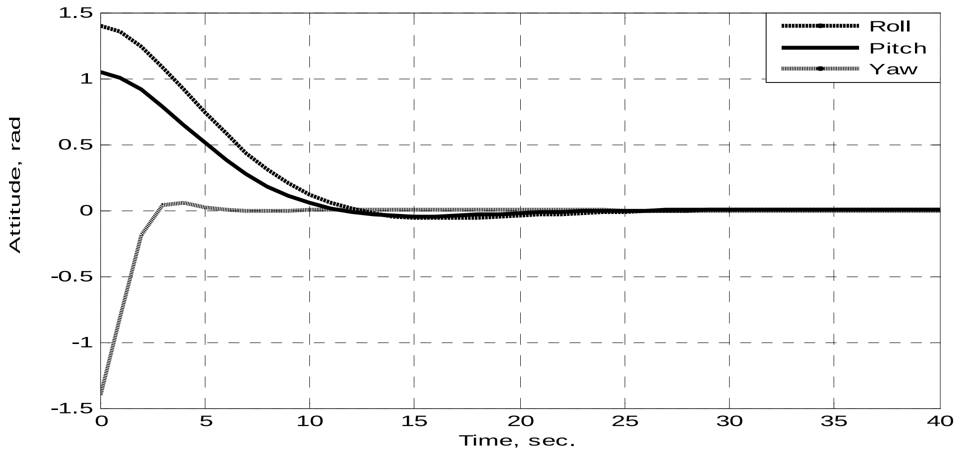

Figure 6: Simulation responses for LQR controller

Discussion of Results

It is observed that the LQR controller is able to damp the oscillations in the system even for large initial attitude displacements within as short period of time as possible. In both Figures 5 and 6, the settling time of the roll axis, pitch axis and yaw axis was about 25sec with a maximum control torque of 1.55E-6Nm. Hence, the higher the angular displacement in the satellite attitude the higher the magnetic control torque needed to damp the oscillations. This also shows the robustness of the controller to attitude parameter changes and also to quickly restore the orientation of the satellite for efficient communication with the base station.

Conclusion and Future Work

The use of active damper to control a gravity gradient stabilised small satellite is presented in this paper. The modelled satellite attitude dynamics is linearised about an equilibrium point and transformed into a state-space equation for the application of an optimal control law-Linear Quadratic Regulator (LQR). As expected, the associated vibration in the gravity gradient stabilised small satellite was dampened within as short a time as possible which shows an autonomous control of small satellite using magnetic torquing only. Further work on closed loop tuning of the weight matrices could reap a better result. It is therefore recommended to use metaheuristic optimisation technique in the choice of weight matrices for future work on similar design.

References

Barry, S. L. (2003). *NPSAT1 Magnetic attitude control system.* 16th Annual AIAA/USU Conference on Small Satellites. Washington, DC. pp.1-7.

Fleeter, R. & Warner, R., (1989). *Guidance and control of miniature satellites, automatic control in aerospace,* IFAC (Tsukuba, Japan), Oxford, UK: Pergamon, pp. 243-248.

Gottfried, K. (2004). *Small satellites: A tool for earth observation?* Invited Paper: Commission IV, Institute of Photogrammetry and GeoInformation, University of Hannover, Hannover

Joseph, M. F. (2006). *LQG/LTR Optimal attitude control of small flexible spacecraft Using free-free boundary conditions.* Unpublished PhD Thesis, Department of Aerospace Engineering Sciences, University of Colorado, USA.

Kleanthis, N., (2007). *Modeling and control of a satellite's geostationary orbit*. Unpublished Master Thesis, Space Science and Technology, Department of Space Science, Kiruna.

Kristin L. M, Andrew J. T. & Christopher D. H. (2001). Design and implementation of a nanosatellite attitude determination and control system. *AAS 01-311*

Lewis, A., (1989). *Instability of gravity gradient spacecraft in full sun orbit: Flight experience from the polar BEAR mission.* 12th Annual AAS Guidance and Control Conf., AAS Paper 89-054, pp. 505-531.

Mohd, N. S. & Varatharajoo, R. (2006). New satellite attitude control structure using the geomagnetic field. *International Journal of Engineering and Technology, 3(2), 175-181.*

Ouhocine, O., Filipski, M. N., Mohd, N. S. B. & Ajir, M. R. (2004). Small satellite attitude control and simulation. *Journal Mekanikal, Bil. 17, 36-47.*

Roland, S. B. (2001). *Advanced control engineering.* Butterworth Heinemann Publishers.

Sidi, M. J. (1997). *Spacecraft dynamics and control: A practical engineering approach.* Cambridge University Press.

Wisniewski, R. (1996). *Satellite attitude control using only electromagnetic actuation.* Unpublished PhD Thesis, Department of Control Engineering Aalborg University.