

OPERATIONS RESEARCH AND LINEAR PROGRAMMING IN APPLICATION OF ASSIGNMENT ALGORITHM

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Abstract

Decision making is very critical and crucial in any organisation, be it private or public. The quality of such decision and ability of such decision to stand the test of time depends on the logic, sense and basis of such decisions. Scientific approach to decision making is therefore central to operations research. This paper presents a general model for the application of assignment algorithm. It provides an insight into the relationship between linear programming and assignment model. It highlights linear programming (LP) assumptions and its limitations which further enhances the assignment technique procedures by means of a matrix. The model used in this paper is Hungarian method. The result indicates the optimum assignment that minimizes total cost.

Keywords: *Operations Research, Linear Programming Assignment Model*

Introduction

A large number of decisions problems faced by a business manager involve allocation of resources to various activities with the objective of increasing profits or decreasing costs or both. The decision problem becomes complicated when a large number of resources are required to be allocated and there are several activities to perform. Rule of thumb, even of an experienced manager, in an likelihood, may not provide the right answer in such cases.

Operations research (OR) is concerned primarily with decision analysis. It is the application of scientific methods, techniques and tools to solve problems involving the operation of a system with optimum solution to the problem (Churchman et al 1957). In the uncertain environment we live in, linear programming (LP) and operations research together help to solve problems such as allocating scarce resources, i.e. allocation of one item only from each source and the assignment of one item only to each location.

Thierauf (1978) defined operations research as that which utilizes the planned approach and interdisciplinary term of personnel to represent complex functional relationships such as mathematical models for the purpose of providing the basis for making decisions and highlight new problems for quantitative analysis.

The application of operations research is to help managers improve upon thier decision making process using various techniques. It sharpens decision making process. In some years past, decision making is based on the rule of thumbs. But operations research provides management with alternative solution to problems and allows judgement based on

sound scientific ground as against decision on the basis of experience and intuition, or put in another, this is how we have been doing it (Ademulegun, 2004). Operations research employs scientific methods for the purpose of solving problems, and there is no place of whims and guess work in it (Vohra 2007). According to him, managerial problems have economic, physical, psychological, biological, sociological and engineering aspect. One of the importance tools used in Operations research is Linear programming.

According to Adamu Idama (1999), operations research is the application of scientific method to the management of organised system. Operations research attempts to provide those who manage organized systems with objective and quantitative basis for decision. Thus, operations research is the application of science to the solution of managerial and administrative problems, and it focuses on the performance of organised systems taken as a whole rather than on thier parts taken separately. It is concerned with how managerial decisions are and should be made. Operations research applies the scientific method to the study of mental work and provides the knowledge and understanding requisite to make effective use of men and machines to carry it out. It provides a decision-maker with scientific basis for decision making.

The general objective of this paper therefore is to present a general model for the application of assignment algorithm. In a more specific term, the aims of the paper are:

- (i) to provide an insight into Linear programming and assignment model.
- (ii) to highligh assignment technique procedure by means of a matrix.
- (iii) to indicate the optimum assignment that minimizes total cost.

Relationship Between Linear Programming and Assignment Model

According to Harper and Lim (1982), the assignment technique is a minimizing technique. It gives a solution that minimizes the values involved. Generally, problems require the following data to be established:

- (a) The objective or goal to be pursued. Such an objective may be:
 - (i) Maximization of profit;
 - (ii) Minimization of cost;
 - (iii) Minimization of time.
- (b) Identification of the various constraints which limit the achievement of the objectives. Such constraints may be classified as follows:
 - (i) Policy: These constraints reflect the various business policies of the organization.
 - (ii) Finance: These constraints usually take the form of budget, standard costs.
 - (iii) Market: The most likely constraint of this nature is the sales limits determined by market demands for the various products.
 - (iv) The production line may be restricted in terms of men, machines, raw materials, floor space or the production methods adopted.

It is usually for a linear programming problem to be subjected to at least two of the above types of two constraints.

- (c) Alternative courses of action must be available to decision maker. This is a pre-requisite condition in a decision making situation as there will be no need to make a decision where there is no option.

The assignment problem can be formulated as a Linear programming model. We first recall that the demand and supply for each is exactly 1 and so we obtain the table.

Table I

	A	B	C	D	Supply
1	10	13	15	18	1
2	14	14	18	10	1
3	11	8	7	16	1
4	18	16	11	9	1
Demand	1	1	1	1	

Source: *Adedayo et al (2006)*.

Suppose we have the $n \times n$ table below:

	1	2	3	j	n
1	a_{11}	a_{21}	a_{1j}	a_{1n}		
2	a_{21}	a_{22}	a_{2j}	a_{2n}		
3	a_{31}	a_{32}	a_{3j}	a_{3n}		
:	:	:	:	:	:		
i	a_{i1}	a_{i2}	a_{ij}	:		
:	a_{ni}	a_{n2}	a_{nj}	a_{nn}		

Let x_{ij} be the decision variable representing assignment of j^{th} job to i^{th} person.

$$\text{Minimize } C = \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_{ij}, \quad i = 1, 2, \dots, n$$

subject to the constraints

$$x_{ij} = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ person is assigned to job } j \text{ otherwise} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{j=1}^n x_{ij} = 1 \quad \text{one job is done by } i^{\text{th}} \text{ person and}$$

$$\sum_{i=1}^n x_{ij} = 1 \quad \text{only one person is assigned to job } j$$

So the assignment problem is formulated as linear programming problem as shown below:

$$\text{Minimize } C = 10X_{11} + 13X_{12} + 15X_{13} + 18X_{14} + 14X_{22} + 18X_{23} + 10X_{24} + 11X_{31} + 8X_{32} + 7X_{33} + 16X_{34} + 18X_{41} + 16X_{42} + 11X_{43} + 9X_{44}$$

$$\text{i.e } C = \sum_{i=1}^4 \sum_{j=1}^4 x_{ij}$$

$$\text{Subject to } x_{11} + x_{12} + x_{13} + x_{14} = 1$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$

$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$

$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$

$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$

$$x_{14} + x_{24} + x_{34} + x_{44} = 1$$

$$x_{ij} = 0 \text{ or } 1$$

If the assignment table has a matrix of order $n \times n$ and also that a customer can be assigned to only one facility and so.

$$\sum_{i=1}^n x_{ij} = \sum_{i=j}^n x_{ij}$$

Linear Programming Assumptions

As with all mathematical models, certain inherent assumptions must be made in order to use Linear programming approach. The modeler must be keenly aware of the impact of these assumptions on the real-life situation being modeled; if the assumptions are deemed unacceptable, the model must be modified or another method developed. According to Lawrence and Pasternack (2002), all Linear models satisfy three assumptions:

- (i) The parameter values are known with certainty.
- (ii) The above function and constraints exhibit constant return to scale.
- (iii) There are no interactions between the decision variables.

In addition to these three assumptions, Linear programming models also require the assumption that the variables are continuous. Speaking in the same vein, Vohra (2007) posited that a Linear programming model is based on the assumptions of proportionality, additivity, continuity, certainty and finite choices.

Harper and Lim (1982) contended that before one can validly use the technique of linear programming, one must ensure that the following assumptions are holding;

- (a) Linearity: There is a linear relationship between the output of each of the following :
 - (i) The total quantity of each resource consumed;
 - (ii) The total receipts;
 - (iii) The total costs.

Such a relationship will hold provided that the selling price, variable costs and resource requirements per unit do not alter over the relevant range of output.

- (b) Divisibility: Perfect divisibility of products and resources exists i.e. products can be produced in fractions and resources employed in the fractions of a unit.
- (c) Additivity: The basic principle of additivity applied throughout the computation.
- (d) Single objectives: The basic problem involves only one objective (e.g. maximization of profit).
- (e) Simple cost function: The total relevant cost function can be divided into fixed and variable elements.
- (f) External factors: All external factors are unchanging.
- (g) Certainty: All quantities and values are known with certainty.

In a similar tune, Daellenbach and Goerge (1978) classified linear programming assumptions under three categories. They called the first assumption divisibility. According to them, all variables can assume any real value. They opined that if the real activity is not infinitely divisible, but the normal activity level is a large number in terms of measurement, then the assumption of divisibility may serve as a convenient approximation. Their final position was that fractional values of the solution are simply rounded to the nearest integer.

The second assumption is non-negativity conditions. This assumption seems to reflect well the nature of most activities in the real world, where it rarely makes sense within an economic or engineering context to talk about negative activity levels.

The third assumption is linearity. Relationships between variables are linear. This is to say that in linear programming this implies.

- (a) Proportionality of contributions: That is the individual contribution of each variable is strictly proportional to its value and the factor of proportionality is constant over the entire range of values that the variable can assume.
- (b) Additivity of contributions: that is the total contribution of all variables is equal to the sum of the individual contributions regardless of the values of the values of the varieties.

Assumption three implies constant returns to scale and precludes economies or diseconomies of scale.

Limitations of Linear Programming

According to Adedayo et al (2006), Linear programming has some limit in its application in practical life situations. Some of these limitations are highlighted below:

- (i) It sometimes gives rise to fractional or decimal solution which is not applicable to problems that require interger solution. For example, a solution is desirable. Another example is when decision is to be taken about whether or not to open a new branch of an office. In such a case, the variable should take values 0 for no and 1 for yes. The Linear programming model will not solve such problems. The proper tool to use for such analysis is the interger programming model.

- (ii) The model assumes that the decision maker has only one goal or objective which may not always be the case since there are situations involving multiple goals by decisions makers. These goals may be equal in dimension, complimentary or even conflicting. The appropriate tool to use in such a situation is the goal programming model.
- (iii) Another limitations lies in the assumptions of linearity. There are situations where the variables are not linearly related to the objective functions or constraints. In such a situation, non Linear programming model is more applicable.
- (iv) The assumption of certainty has its limitation in that some situations occur where factors like costs, constraint requirement may not be known. In such situations, we use the probabilistic Linear programming model.

Harper and Lim (1982) were of the view that it should be fully appreciated that a given linear programming solution may not in the end prove valid, for the following reasons:

- (a) The basic assumption that all the variables are linear may not necessarily linear and so linear programming cannot be used in such situations.
- (b) The assumption of divisibility may not be justified. Often fractions of a unit cannot be produced and sold
- (c) The assumption of certainty requires that uncertain variables are represented by point estimates. This could prove to be misleading.
- (d) The solution may not be optimal if the input cost and prices used do not represent the external opportunity cost of the fractions involved. Optimizing technique with respect to linear programming means a situation in which the variables give straight line curves on graph which in this context usually means if one doubles the quantity one doubles the variable cost or contribution.
- (e) The time factor is ignored. Whether a cost is variable or fixed depends on range and this is completely in linear programming.
- (f) Many units of an input factor are assumed to be bought at one price only. As a result, the solution will fail to account of potential quantity discounts, possible increased prices as suppliers diminish.
- (g) Some costs rise stepwise rather than linearly.
- (h) The assumption that each unit of output requires constant amount of each input factor may not be realistic. There could well be products that need a "warming-up" period, for example, before their assumed unit production timer are reached.
- (i) Existence of intangible costs and benefits which are not readily quantifiable may not be incorporated in the problem formulation (e.g. goodwill).
- (j) Since the procedure result in a solution that is optimal for the available input data than any opportunities that are not included in the formulation will be ignored.
- (k) The constraints are not as rigid as is implied by the theory (e.g. a small organization change virtually no cost could result in a few more hours being made available).

Assignments Technique as a Minimizing Technique

Assignment is a problem because people possess varying abilities for performing different jobs and, therefore, the costs of performing those jobs by different people are different. Obviously, if all persons could do a job in the same time, or at the same cost then it would not matter who among them is assigned the job. Thus, in an assignment problem, the question is how the assignment should be made in order that the total cost involved is minimized (Vohra 2007).

Assignment problem is a variation of the transportation problem with two characteristics. First, the pay-off matrix for the problem would be a square matrix, and second, the optimal solution to the problem would always be such that there would be only one assignment in a given row or column of the pay-off matrix.

Ray Wild (1986) stated that assignment problem is a special case of the transportation problem. He was of the view that in the assignment problem we are concerned with the allocation of one item only from each source and the assignment of one item only to each location.

Assignment of objects are done on one-to-one basis with finite number of objects to be assigned to finite resources. Also, the tasks must be assigned to facilities on one-to-one basis. In other words, no one task must be assigned to two facilities and no one facility must be assigned to perform two tasks. Each assignment problem uses a table. The numbers in the table represent the costs or times associated with each particular assignment. If there are n objects to be matched, there is $n!$ different matches and when the number is high, it becomes tedious. The measure of effectiveness of each job for each machine can be represented by means of a matrix.

Assignment Technique Procedure

Ray Wild (1986) identified fourteen procedures in assignment technique. A solution to this type of problem according to him can be obtained by means of the simple routine described below:

1. Take out the minimum figure from each row.
2. Deduct each minimum figure from the figure in that row.
3. Determine the least number vertical and/or horizontal lines required to cover all zeroes in the new matrix.
4. If the number of lines is less than the number of columns or rows proceed to the next step.
5. Take the minimum figures from each of the new columns.
6. Deduct each minimum figure from each of the figures in that column.
7. Determine the least number of vertical and/or horizontal lines required to cover all zeroes in the new matrix.
8. If the number of lines is less than the number of columns or rows.
9. Identify minimum uncovered element in the new matrix.
10. (a) Subtract this number from all uncovered elements in the new matrix.
(b) Add this number to those elements covered by two lines.

- (c) Do not change those elements covered by one line.
11. Determine the least number of lines to cover all zeroes.
 12. If the number of lines is less than the number of columns or rows, repeat step 9, 10, and 11 until the number of lines is equal to the number of columns or rows.
 13. The optimal assignment is obtained when the number of lines equals the number of columns or rows. The assignment of customers to facility is given by the zeroes in the matrix.
 14. The cost associated with the optimal assignment may then be calculated.

Facilities	Customers					
	A	B	C	D	E	
	1	10	5	9	10	11
	2	13	19	6	12	14
	3	3	2	4	4	5
	4	18	9	12	17	15
	5	11	6	14	19	10

Source: *Adopted from Ray Wild (1986)*

Step 1						
	A	B	C	D	E	ROW MIN.
1	10	5	9	10	11	5
2	13	19	6	12	14	6
3	3	2	4	4	5	2
4	18	9	12	17	15	9
5	11	6	14	19	10	6

Source: *Researcher's Computation (2012)*

In the above table, the minimum value in each row is determined. The minimum value in row 1 is 5, row 2 is 6, row 3 is 2, row, 4 is 9 and row 5 is 6.

When the minimum value in each row is subtracted from the value in each row, the result gives the row opportunity cost table.

Row Opportunity Cost table					
	A	B	C	D	E
1	5	0	4	5	6
2	7	13	0	6	8
3	1	0	2	2	3
4	9	0	3	8	6
5	5	0	8	13	4

Source: *Researcher's Computation (2012)*

Step 3					
	A	B	C	D	E
1	5	0	4	5	6
2	7	13	0	6	8
3	1	0	2	2	3
4	9	0	3	8	6
5	5	0	8	13	4

Source: Researcher's Computation (2012)

In step 3 above, the minimum of horizontal or vertical lines are drawn to cross all zeros. If the number of such lines equals the number of row or column, then an optimal assignment exists. In step 3 above, the minimum number of vertical lines drawn are 2, whereas, there are 5 rows and 5 columns. Since the numbers of vertical lines drawn are not equal to 5, then we go to the next step by computing column opportunity cost table.

Step 4					
	A	B	C	D	E
1	5	0	4	5	6
2	7	13	0	6	8
3	1	0	2	2	3
4	9	0	3	8	6
5	5	0	8	13	4
Column Min.	1	0	0	2	3

Source: Researcher's Computation (2012)

In step 4 above, the minimum value in each column is determined. The minimum value in column 1 is 1, column 2 is zero, column 3 is zero, column 4 is 2 and column 5 is 3.

Step 5 and 6 Opportunity Cost Table					
	A	B	C	D	E
1	4	0	4	3	3
2	6	13	0	4	5
3	0	0	2	0	0
4	8	0	3	6	3
5	4	0	8	11	1

Source: Researcher's Computation (2012)

Step 5 and 6 above are obtained by deducting the minimum value in each column from the value in each column.

Step 7 and 8

	A	B	C	D	E
1	4	0	4	3	3
2	6	13	0	4	5
3	0	0	2	0	0
4	8	0	3	6	3
5	4	0	8	11	1

Source: Researcher's Computation (2012)

Step 7 and 8 above are obtained by drawing the minimum number of vertical or horizontal lines to cross all zeros. If the number equals the number of row or column, then, optimal assignment is reached. But in the above case, the number of vertical and or horizontal lines drawn is not equal to 5, then proceeds to step 9 and 10 by subtracting the minimum uncrossed value or estimate from an uncrossed cells and add this to all covered elements at the intersection of two lines. The result of this is table 9 and 10 below.

Step 9 and 10

	A	B	C	D	E
1	3	0	4	2	2
2	5	13	0	3	4
3	0	1	3	0	0
4	8	0	3	5	2
5	3	0	8	10	0

Source: Researcher's Computation (2012)

Step 11

	A	B	C	D	E
1	3	0	4	2	2
2	5	13	0	3	4
3	0	1	3	0	0
4	8	0	3	5	2
5	3	0	8	10	0

Source: Researcher's Computation (2012)

Step 11 above is obtained by drawing the minimum number of vertical or horizontal lines to cross all zeros. If the numbers of such lines drawn are equal to 5, then the optimal assignment is reached. But in the above, the number of vertical and horizontal lines drawn to cross all zeros is not equal to 5. Then proceed to the next stage by subtracting the minimum, uncrossed value or element and add this to all uncovered elements at the intersection of two lines. This will give us the new matrix in step 12 and 13.

Step 12 and 13

	A	B	C	D	E
1	1	0	4	0	0
2	3	13	0	3	4
3	0	3	5	0	0
4	5	0	3	3	0
5	3	2	10	10	0

Source: Researcher's Computation (2012)

In step 12 and 13 above, the minimum number of vertical and or horizontal lines drawn are equal to 5 which equal the number of row and column. It implies that optimum assignment has been reached.

	A	B	C	D	E
1	1	0	4	0	0
2	3	13	0	3	4
3	0	3	5	0	0
4	5	0	3	3	0
5	3	2	10	10	0

Source: Researcher's Computation (2012)

In the above table, we allocate to table with zero opportunity cost.

Step 14

Allocation Cost from initial Matrix

#,000

1-----	D	-----10
2-----	C	-----6
3-----	A	-----3
4-----	B	-----9
5-----	E	-----10

#39

The third objective of this paper is to indicate the optimum assignment that minimizes the total cost. The optimum assignment that will achieve this objective is to allocate, facility 1 to customer D. The associated cost is #10,000. Facility 2 customer C with the associated cost of #6,000. Facility 3 to customer A with associated cost of #3,000. Facility 4 to customer B with associated cost of #9,000, and facility 5 to customer E with associated cost of #10,000.

Conclusion and Recommendations

The complexity and dynamics of business world makes sound, logical and scientific approach to decision making desirable. The world economy is transforming beyond the ignorance of the past by managers to take decision without understanding the results and implications of

such decisions. It is no longer admissible for managers to take decisions based on intuitive and doubts.

The application of operations research is to help managers sharpen and improve upon their decision making process using various available techniques. Linear programming is one of the widely used managerial techniques developed to fast track decision making process. It adopts a team approach to problem solving using multidisciplinary members. There is always interaction with and involvement of decision makers.

Linear programming formulations have proven quite valuable for solving numerous problems in business and government. The application of linear programming is extensive and includes the allocation of man, machine, money and materials for the most productive use. Other areas of use of Linear programming include product mix, crop mix, direct mix, capital rationing decisions, transportation, assignment problems and replacement policy just to mention a few.

Linear programming and assignment problem may not necessarily be panacea or solution to all management problems, but they provide scientific framework for reliable, and sound decision making in the face of various constraints and uncertainties.

Assignment problem involves the optimal allocation of various productive resources having different efficiencies to various tasks that are to be completed. The object of any assignment problem is minimization of costs. Given an assignment problem, a unique solution can be derived by using Hungarian method because it is based on a theorem proved by Dr. Kong, a Hungarian mathematician.

Inspite of various limitations of operations research and linear programming to solve complex organizational problem, the approach remains a very veritable strategy to enhance efficiency and effectiveness in decision making.

To attack operational problems in a complex, dynamic and uncertain business and economic environment, the application of operations research, linear programming and assignment problem is therefore not only necessary, but desirable to enhance better decision making.

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