INTEGRATED COLLOCATION METHODS FOR SOLVING FOURTH ORDER INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract
Two numerical methods for solving fourth order Integro-differential equations are discussed in this paper. The methods are Integrated Standard Collocation Method and Integrated Perturbed Collocation Method. The methods are based on replacement of the highest derivative that appeared in the problem considered by Power series and Chebyshev polynomials of appropriate degree. After simplification, we then collocate the resulting equation at some equally spaced interior points in the interval of consideration. This results into system of linear algebraic equations which are then solved by Gaussian elimination method to obtain the values of the unknown constants that appeared in the assumed solution. Numerical experiments show that the methods are easy to apply and of high accuracy. From the results obtained, we observe that the Integrated Standard Collocation Method by the two bases functions perform better than the Integrated Perturbed Collocation Method. Also, the Integrated Standard Collocation Method involved lower order matrix than the Integrated Perturbed Collocation Method.

Keywords: Integrated Collocation Method, Power series, Chebyshev polynomials, Perturbation, Integro-Differential Equations

Introduction
The numerical solutions of functional equations such as differential equations, integral equations and Integro-Differential Equations is a field of in which active research work is currently going on. Integro-Differential Equations occur in many areas including astronomy, potential theory, fluid dynamics, chemical kinetics and biological reactions (Taiwo, Jimoh & Bello, 2014).

Most Integro-Differential Equations defy analytical approach to obtain their closed form solutions (Taiwo & Onumanyi, 1991). Therefore, the need to adopt numerical techniques in order to obtain approximate solutions to such problems cannot be over emphasized (Taiwo, 2005; Mohamed & Khader, 2011; Sweilam, Khader & Kota, 2011; Khader & Hendy, 2012). Recently, several authors have investigated the numerical solutions of fourth order Integro-Differential Equations, among which include spline functions expansion (Asady & Kajani, 2005), Legendre pseudo-spectral method (Mustapha, 2008) and Tau numerical solution with arbitrary polynomial base (El-Sayed & Abdel-Aziz, 2003).

For the purpose of our discussion, we shall consider the general fourth order linear and non-linear Integro-Differential Equations of the following types:

(i) FredholmIntegro-Differential Equations (FIDE)

$$\sum_{i=0}^{4} P_i y^{(i)}(x) + \int_a^b k(x,t)y(t)dt = f(x) \quad (1)$$

(ii) VolterraIntegro-Differential Equations (VIDE)

$$\sum_{i=0}^{4} P_i y^{(i)}(x) + \int_a^x k(x,t)y(t)dt = f(x) \quad (2)$$

Here, equations (1) and (2) are subjected to the conditions

$$\alpha_1 y(a) + \alpha_2 y'(a) + \alpha_3 y''(a) + \alpha_4 y'''(a) = \gamma_1 \quad (3)$$

and

$$\beta_1 y(b) + \beta_2 y'(b) + \beta_3 y''(b) + \beta_4 y'''(b) = \gamma_2 \quad (4)$$

where, $P_i (i \geq 0)$ are constants, $k(x,t)$ and $f(x)$ are given smooth (i.e. differentiable and integrable) functions in $[a,b]$, $y^{(i)}(x)$ denotes the ith derivative of $y(x)$, α_i, β_i, γ_1 and γ_2 are real finite constants and $y(x)$ is the unknown function to be determined.

Methodology and Techniques

In this section, we shall consider the following methods for the solutions of equations (1) - (4).

(i) Integrated Standard Collocation Method, and
(ii) Integrated Perturbed Collocation Method.

To illustrate the basic concept of the Integrated Collocation Method, we consider the following general non-linear system:

$$L[y(x)] + N[y(x)] = f(x) \quad (5)$$

where, L is a linear operator, N is a non-linear operator and $f(x)$ is a given smooth function.

For non-linear problem, we employ the Taylor's series linearization scheme to obtain a linear approximation at $t_0 = 0$.

Taylor's series linearization scheme

Let

$$G_y = y^{(n)}(t) \quad (6)$$

be the non-linear part of equation (1), expanding the right hand side of equation (6) in Taylor's series around the point t_0, we obtain

$$G_y \equiv y(t) + (t - t_0)y'(t) + \frac{(t-t_0)^2 y''(t)}{2!} + \frac{(t-t_0)^3 y'''(t)}{3!} + \ldots + \frac{(t-t_0)^n y^{(n)}(\varphi)}{n!} \quad (7)$$

Putting $t_0 = 0$ in equation (7), we have

$$G_y \equiv y(t) + ty'(t) + \frac{t^2}{2!} y''(t) + \frac{t^3}{3!} y'''(t) + \ldots + \frac{t^n}{n!} y^{(n)}(\varphi) \quad (8)$$

Truncating equation (8) at the term containing $y'(t)$, we have

$$G_y \approx y(t) + ty'(t) \quad (9)$$

Therefore, equation (9) is a linear approximation to equation (6).

Chebyshev Polynomials

The Chebyshev polynomials of degree n of the first kind which is valid in the interval $-1 \le x \le 1$ is defined

$$T_n(x) = \cos[n \cos^{-1} x], \qquad n \ge 0 \quad (10)$$

where,

$$T_0(x) = 1$$

$$T_1(x) = x$$

and the recurrence relation of equation (10) is given by

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \qquad n \ge 1. \quad (11)$$

For interval $[a,b]$, we have

$$T_n(x) = \cos[n \cos^{-1}(\frac{2x-a-b}{b-a})], \qquad a \le x \le b \quad (12)$$

and this is satisfied by the recurrence relation

$$T_{n+1}(x) = 2(\frac{2x-a-b}{b-a})T_n(x) - T_{n-1}(x),\ n \ge 1,\ a \le x \le b \quad (13)$$

Integrated Standard Collocation Method by Power Series Approach (ISCMPS)

In order to apply this method to solve equation (1) together with the initial conditions given in equations (3) and (4), we assumed the power series solution given by

$$\frac{d^4 y}{dx^4} = \sum_{i=0}^{N} a_i x^i \quad (14)$$

and

$$\frac{d^3 y}{dx^3} = \int \sum_{i=0}^{N} a_i x^i dx + c_1 \quad (15)$$

$$\frac{d^2 y}{dx^2} = \int\int \sum_{i=0}^{N} a_i x^i dxdx + c_1 x + c_2 \quad (16)$$

$$\frac{dy}{dx} = \int\int\int \sum_{i=o}^{N} a_i x^i dxdxdx + \frac{c_1}{2} x^2 + c_2 x + c_3 \quad (17)$$

$$y(x) = \iiiint \sum_{i=0}^{N} a_i x^i dxdxdxdx + \frac{c_1}{6}x^3 + \frac{c_2}{2}x^2 + c_3 x + c_4 \quad (18)$$

Equation (1) is re-written as

$$P_0 y(x) + P_1 y'(x) + P_2 y''(x) + P_3 y'''(x) + P_4 y^{iv}(x) + \int_a^b k(x,t)y(t)dt = f(x) \quad (19)$$

Substituting equations (14) – (18) into (19), we obtain

$$P_0[\iiiint \sum_{i=0}^{N} a_i x^i dxdxdxdx + \frac{c_1}{6}x^3 + \frac{c_2}{2}x^2 + c_3 x + c_4] + P_1[\iiint \sum_{i=o}^{N} a_i x^i dxdxdx + \frac{c_1}{2}x^2 + c_2 x + c_3]$$
$$+ P_2[\iint \sum_{i=0}^{N} a_i x^i dxdx + c_1 x + c_2] + P_3[\int \sum_{i=0}^{N} a_i x^i dx + c_1] + P_4 \sum_{i=0}^{N} a_i x^i + \int_a^b k(x,t)y(t)dt = f(x) \quad (20)$$

where,

$$y(t) = \iiiint \sum_{i=0}^{N} a_i t^i dtdtdtdt + \frac{c_1}{6}t^3 + \frac{c_2}{2}t^2 + c_3 t + c_4$$

Expanding and simplifying equation (16), we obtain

$$\begin{aligned}
&P_0[a_0 W_0^{(4)}(x) + a_1 W_1^{(4)}(x) + a_2 W_2^{(4)}(x) + ... + a_N W_N^{(4)}(x)] \\
&+ P_1[a_0 W_0^{(3)}(x) + a_1 W_1^{(3)}(x) + a_2 W_2^{(3)}(x) + ... + a_N W_N^{(3)}(x)] \\
&+ P_2[a_0 W_0^{(2)}(x) + a_1 W_1^{(2)}(x) + a_2 W_2^{(2)}(x) + ... + a_N W_N^{(2)}(x)] \\
&+ P_3[a_0 W_0^{(1)}(x) + a_1 W_1^{(1)}(x) + a_2 W_2^{(1)}(x) + ... + a_N W_N^{(1)}(x)] \\
&+ P_4[a_0 W_0^{(0)}(x) + a_1 W_1^{(0)}(x) + a_2 W_2^{(0)}(x) + ... + a_N W_N^{(0)}(x)] \\
&+ C(X) + G_1(X,t) = f(x). \quad (21)
\end{aligned}$$

Here,

$$W_N^{(4)}(x) = \iiiint x^N dxdxdxdx \quad (22)$$

$$W_0^{(4)}(x) = \iiiint dxdxdxdx \quad (23)$$

$$C(X) = P_0(\frac{c_1}{6}x^3 + \frac{c_2}{2}x^2 + c_3 x + c_4) + P_1(\frac{c_1}{2}x^2 + c_2 x + c_3) + P_2(c_1 x + c_2) + P_3 c_1 \quad (24)$$

and

$$G_1(X,t) = \int_a^b k(x,t)y(t)dt \quad (25)$$

After evaluating the terms involving integrals in equation (25) and with further simplification, we then collocate the left-over at the point $x = x_z$, we obtain

$$\begin{aligned}
&P_0[a_0 W_0^{(4)}(x_z) + a_1 W_1^{(4)}(x_z) + a_2 W_2^{(4)}(x_z) + ... + a_N W_N^{(4)}(x_z)] \\
&+ P_1[a_0 W_0^{(3)}(x_z) + a_1 W_1^{(3)}(x_z) + a_2 W_2^{(3)}(x_z) + ... + a_N W_N^{(3)}(x_z)] \\
&+ P_2[a_0 W_0^{(2)}(x_z) + a_1 W_1^{(2)}(x_z) + a_2 W_2^{(2)}(x_z) + ... + a_N W_N^{(2)}(x_z)] \\
&+ P_3[a_0 W_0^{(1)}(x_z) + a_1 W_1^{(1)}(x_z) + a_2 W_2^{(1)}(x_z) + ... + a_N W_N^{(1)}(x_z)] \\
&+ P_4[a_0 W_0^{(0)}(x_z) + a_1 W_1^{(0)}(x_z) + a_2 W_2^{(0)}(x_z) + ... + a_N W_N^{(0)}(x_z)] \\
&+ C(X_z) + G_1(X_z,t) = f(x_z). \quad (26)
\end{aligned}$$

where,

$$x_z = a + \frac{(b-a)z}{N+2},\ z = 1,2,3,.......N+1 \quad (27)$$

Putting equation (27) into (26), we obtain (N+1) algebraic equations with (N+5) unknown constants. Four extra equations are obtained using the initial conditions given in equations (3) and (4). Altogether, we have (N+5) algebraic equations with (N+5) unknown constants. This system of (N+5) algebraic linear equations is put in vector form as $A\underline{X} = \underline{b}$ and then solved using Gaussian elimination method to obtain the unknown constants $a_i (i \geq 0)$ and $c_i's$. These values are then substituted into our assumed solution to obtain the approximate solution.

Integrated Perturbed Collocation Method by Power Series Approach (IPCMPS)

Again, we apply the Integrated Perturbed Collocation method using Power series as our basis function to solve equation (1) together with the initial conditions given in equations (3) and (4).

We assume the solution of the form

$$\frac{d^4y}{dx^4} = \sum_{i=0}^{N} a_i x^i + H_N(x) \quad (28)$$

Integrating equation (28) successively, we obtain

$$\frac{d^3y}{dx^3} = \int \sum_{i=0}^{N} a_i x^i dx + \int H_N(x)dx + c_1 \quad (29)$$

$$\frac{d^2y}{dx^2} = \int\int \sum_{i=0}^{N} a_i x^i dxdx + \int\int H_N(x)dxdx + c_1x + c_2 \quad (30)$$

$$\frac{dy}{dx} = \int\int\int \sum_{i=o}^{N} a_i x^i dxdxdx + \int\int\int H_N(x)dxdxdx + \frac{c_1}{2}x^2 + c_2x + c_3 \quad (31)$$

$$y(x) = \int\int\int\int \sum_{i=0}^{N} a_i x^i dxdxdxdx + \int\int\int\int H_N(x)dxdxdxdx + \frac{c_1}{6}x^3 + \frac{c_2}{2}x^2 + c_3x + c_4 \quad (32)$$

where,

$$H_N(x) = \tau_1 T_N(x) + \tau_2 T_{N-1}(x) + \tau_3 T_{N-2}(x) + \tau_4 T_{N-3}(x) \quad (33)$$

$\tau_i (i = 1,2,3,4)$ are four free tau parameters to be determined along with the constants $a_i's$ and $T_N(x)$ is the Chebyshev polynomials of degree N as discussed in section (2.2).

Substituting equations (28) – (32) into equation (19), we obtain

$$P_0[\int\int\int\int \sum_{i=0}^{N} a_i x^i dxdxdxdx + \int\int\int\int H_N(x)dxdxdxdx + \frac{c_1}{6}x^3 + \frac{c_2}{2}x^2 + c_3x + c_4]$$

$$+ P_1[\int\int\int \sum_{i=o}^{N} a_i x^i dxdxdx + \int\int\int H_N(x)dxdxdx + \frac{c_1}{2}x^2 + c_2x + c_3]$$

$$+ P_2[\int\int \sum_{i=0}^{N} a_i x^i dxdx + \int\int H_N(x)dxdx + c_1x + c_2] + P_3[\int \sum_{i=0}^{N} a_i x^i dx + \int H_N(x)dx + c_1]$$

$$+ P_4[\sum_{i=0}^{N} a_i x^i + H_N(x)] + \int_a^b k(x,t)y(t)dt = f(x) \quad (34)$$

Expanding and simplifying equation (34), we obtain

$$
\begin{aligned}
&P_0[a_0W_0^{(4)}(x)+a_1W_1^{(4)}(x)+a_2W_2^{(4)}(x)+...+a_NW_N^{(4)}(x)]\\
&+P_1[a_0W_0^{(3)}(x)+a_1W_1^{(3)}(x)+a_2W_2^{(3)}(x)+...+a_NW_N^{(3)}(x)]\\
&+P_2[a_0W_0^{(2)}(x)+a_1W_1^{(2)}(x)+a_2W_2^{(2)}(x)+...+a_NW_N^{(2)}(x)]\\
&+P_3[a_0W_0^{(1)}(x)+a_1W_1^{(1)}(x)+a_2W_2^{(1)}(x)+...+a_NW_N^{(1)}(x)]\\
&+P_4[a_0W_0^{(0)}(x)+a_1W_1^{(0)}(x)+a_2W_2^{(0)}(x)+...+a_NW_N^{(0)}(x)]\\
&\quad+H_N^{(P_i)}(X)+C(X)+G_1(X,t)=f(x)\,. \qquad (35)
\end{aligned}
$$

Here,

$$
\begin{aligned}
H_N^{(P_i)}(X)&=P_0\int\int\int\int H_N(x)dxdxdxdx+P_1\int\int\int H_N(x)dxdxdx\\
&+P_2\int\int H_N(x)dxdx+P_3\int H_N(x)dx+P_4H_N(x) \qquad (36)
\end{aligned}
$$

After evaluating the terms involving integrals in equation (35) and with further simplification, we then collocate the left-over at the point $x=x_z$, we obtain

$$
\begin{aligned}
&P_0[a_0W_0^{(4)}(x_z)+a_1W_1^{(4)}(x_z)+a_2W_2^{(4)}(x_z)+...+a_NW_N^{(4)}(x_z)]\\
&+P_1[a_0W_0^{(3)}(x_z)+a_1W_1^{(3)}(x_z)+a_2W_2^{(3)}(x_z)+...+a_NW_N^{(3)}(x_z)]\\
&+P_2[a_0W_0^{(2)}(x_z)+a_1W_1^{(2)}(x_z)+a_2W_2^{(2)}(x_z)+...+a_NW_N^{(2)}(x_z)]\\
&+P_3[a_0W_0^{(1)}(x_z)+a_1W_1^{(1)}(x_z)+a_2W_2^{(1)}(x_z)+...+a_NW_N^{(1)}(x_z)]\\
&+P_4[a_0W_0^{(0)}(x_z)+a_1W_1^{(0)}(x_z)+a_2W_2^{(0)}(x_z)+...+a_NW_N^{(0)}(x_z)]\\
&\quad H_N^{(P_i)}(X_z)+C(X_z)+G_1(X_z,t)=f(x_z)\,. \qquad (37)
\end{aligned}
$$

where,

$$
x_z=a+\frac{(b-a)z}{N+6},\ z=1,2,3,.......N+5 \qquad (38)
$$

Putting equation (38) into (37), we obtain (N+5) algebraic equations with (N+9) unknown constants. Four extra equations are obtained using the initial conditions given in equations (3) and (4). Altogether, we have (N+9) algebraic equations with (N+9) unknown constants. This system of (N+9) algebraic linear equations is put in vector form as $A\underline{X}=\underline{b}$ and then solved using Gaussian elimination method to obtain the unknown constants $a_i\,(i\geq 0)$ and $c_i's$ and the parameters $\tau_i's$. These values are then substituted into our assumed solution to obtain the approximate solution.

Remarks: Using Chebyshev Polynomials as bases functions

We replace the Power series in the assumed solutions given by equations (14) and (28) by their Chebyshev polynomial equivalents for the two methods discussed in this article.

Numerical Examples

In this section, we demonstrate both the Integrated Standard Collocation and Integrated Perturbed Collocation Methods to solve some linear and non-linear fourth order Integro-differential equations. This class of problems commonly appear in many physical and biological applications such as heat transfer, diffusion process and biological species coexisting together with increasing and decreasing rates of generating.

Remark: We define error used as

$$Error = | y(x) - y_N(x) | : a \le x \le b \text{ for } N = 1,2,3,..........$$

Example 1:Consider the fourth order linear VolterraIntegro-differential equation

$$y^{iv}(x) - y(x) + \int_0^x y(t)dt = x + (x+3)e^x \ , \qquad 0 \le x \le 1. \qquad (39)$$

subject to the boundary conditions

$$y(0) = 1, \quad y(1) = 1 + e, \qquad y''(0) = 2, \qquad y''(1) = 3e.$$

The exact solution of this problem is

$$y(x) = 1 + xe^x$$

Example 2: Consider the fourth order linear FredholmIntegro-differential equation

$$y^{iv}(x) - 2y''(x) - \int_0^1 xy(t)dt = 3x, \qquad 0 \le x \le 1. \qquad (40)$$

subject to the boundary conditions

$$y(0) = 0, \quad y(1) = 1, \qquad y''(0) = 0, \qquad y''(1) = -\frac{84}{47}.$$

The exact solution of this problem is

$$y(x) = -\frac{14}{47}x^3 + \frac{61}{47}x.$$

Example 3: Consider the fourth order non-linear VolterraIntegro-differential equation

$$y^{iv}(x) - \int_0^x e^{-t} y^2(t)dt = 1, \qquad 0 \le x \le 1. \qquad (41)$$

subject to the boundary conditions

$$y(0) = 1, \quad y(1) = e, \qquad y''(0) = 1, \qquad y''(1) = e..$$

The exact solution of this problem is

$$y(x) = e^x.$$

Tables of Results

Table 1a: Numerical Results for Example 1

x	Exact	Integrated Standard Collocation Method			
		Power Series		Chebyshev Polynomials	
		N=4	Error	N=4	Error
0.0	1.00000	1.00000	0.00000	1.00000	0.00000
0.1	1.11052	1.11053	1.00E-5	1.11053	1.00E-5
0.2	1.24428	1.24429	1.00E-5	1.24429	1.00E-5
0.3	1.40496	1.40497	1.00E-5	1.40497	1.00E-5
0.4	1.59673	1.59674	1.00E-5	1.59674	1.00E-5
0.5	1.82436	1.82437	1.00E-5	1.82437	1.00E-5
0.6	2.09327	2.09326	1.00E-5	2.09326	1.00E-5
0.7	2.40963	2.40962	1.00E-5	2.40962	1.00E-5
0.8	2.78043	2.78042	1.00E-5	2.78042	1.00E-5
0.9	3.21364	3.21363	1.00E-5	3.21363	1.00E-5
1.0	3.71828	3.71828	0.00000	3.71828	0.00000

Table 1b: Numerical Results for Example 1

x	Exact	Integrated Perturbed Collocation Method			
		Power Series		Chebyshev Polynomials	
		N=4	Error	N=4	Error
0.0	1.00000	1.00000	0.00000	1.00000	0.00000
0.1	1.11052	1.11053	1.00E-5	1.11053	1.00E-5
0.2	1.24428	1.24429	1.00E-5	1.24429	1.00E-5
0.3	1.40496	1.40497	1.00E-5	1.40497	1.00E-5
0.4	1.59673	1.59674	1.00E-5	1.59674	1.00E-5
0.5	1.82436	1.82437	1.00E-5	1.82437	1.00E-5
0.6	2.09327	2.09326	1.00E-5	2.09326	1.00E-5
0.7	2.40963	2.40962	1.00E-5	2.40962	1.00E-5
0.8	2.78043	2.78042	1.00E-5	2.78042	1.00E-5
0.9	3.21364	3.21363	1.00E-5	3.21363	1.00E-5
1.0	3.71828	3.71828	0.00000	3.71828	0.00000

Table 2a: Numerical Results for Example 2

x	Exact	Integrated Standard Collocation Method			
		Power Series		Chebyshev Polynomials	
		N=4	Error	N=4	Error
0.0	0.000000	0.000000	0.000000	0.000000	0.000000
0.1	0.129489	0.129489	0.000000	0.129489	0.000000
0.2	0.257191	0.257191	0.000000	0.257191	0.000000
0.3	0.381319	0.381318	1.000E-6	0.381318	1.000E-6
0.4	0.500085	0.500084	1.000E-6	0.500084	1.000E-6
0.5	0.611702	0.611701	1.000E-6	0.611701	1.000E-6
0.6	0.714383	0.714382	1.000E-6	0.714382	1.000E-6
0.7	0.806340	0.806339	1.000E-6	0.806339	1.000E-6
0.8	0.885787	0.885785	2.000E-6	0.885786	1.000E-6
0.9	0.950936	0.950934	2.000E-6	0.950935	1.000E-6
1.0	1.000000	0.999997	3.000E-6	0.999999	1.000E-6

Table 2b: Numerical Results for Example 2

x	Exact	Integrated Perturbed Collocation Method			
		Power Series		Chebyshev Polynomials	
		N=4	Error	N=4	Error
0.0	0.000000	0.000000	0.000000	0.000000	0.000000
0.1	0.129489	0.129489	0.000000	0.129489	0.000000
0.2	0.257191	0.257191	0.000000	0.257191	0.000000
0.3	0.381319	0.381318	1.000E-6	0.381318	1.000E-6
0.4	0.500085	0.500084	1.000E-6	0.500084	1.000E-6
0.5	0.611702	0.611701	1.000E-6	0.611700	2.000E-6
0.6	0.714383	0.714382	1.000E-6	0.714381	2.000E-6
0.7	0.806340	0.806339	1.000E-6	0.806337	3.000E-6
0.8	0.885787	0.885785	2.000E-6	0.885782	5.000E-6
0.9	0.950936	0.950934	2.000E-6	0.950929	7.000E-6
1.0	1.000000	0.999997	3.000E-6	0.999990	1.000E-5

Table 3a: Numerical Results for Example 3

x	Exact	Integrated Standard Collocation Method			
		Power Series		Chebyshev Polynomials	
		N=4	Error	N=4	Error
0.0	1.00000	1.00000	0.00000	1.00000	0.00000
0.1	1.10517	1.10500	1.70E-4	1.10502	1.50E-4
0.2	1.22140	1.22107	3.30E-4	1.22112	2.80E-4
0.3	1.34986	1.34938	4.80E-4	1.34946	4.00E-4
0.4	1.49182	1.49124	5.80E-4	1.49134	4.80E-4
0.5	1.64872	1.64806	6.60E-4	1.64818	5.40E-4
0.6	1.82212	1.82143	6.90E-4	1.82158	5.40E-4
0.7	2.01375	2.01308	6.70E-4	2.01327	4.80E-4
0.8	2.22554	2.22495	5.80E-4	2.22516	3.80E-4
0.9	2.45960	2.45913	4.70E-4	2.45938	2.20E-4
1.0	2.71828	2.71796	3.20E-4	2.71824	4.00E-5

Table 3b: Numerical Results for Example 3

x	Exact	Integrated Perturbed Collocation Method			
		Power Series		Chebyshev Polynomials	
		N=4	Error	N=4	Error
0.0	1.00000	1.00000	0.00000	1.00000	0.00000
0.1	1.10517	1.10502	1.50E-4	1.10502	1.50E-4
0.2	1.22140	1.22112	2.80E-4	1.22112	2.80E-4
0.3	1.34986	1.34946	4.00E-4	1.34946	4.00E-4
0.4	1.49182	1.49134	4.80E-4	1.49134	4.80E-4
0.5	1.64872	1.64819	5.30E-4	1.64818	5.40E-4
0.6	1.82212	1.82158	5.40E-4	1.82158	5.40E-4
0.7	2.01375	2.01327	4.80E-4	2.01327	4.80E-4
0.8	2.22554	2.22516	3.80E-4	2.22516	3.80E-4
0.9	2.45960	2.45938	2.20E-4	2.45938	2.20E-4
1.0	2.71828	2.71824	4.00E-5	2.71824	4.00E-5

Discussion of Results and Conclusion

In this paper, we have been able to show that both the Power Series and Chebyshev Polynomial Integrated Collocation methods can efficiently solve linear and non-linear fourth-order integro-differential equations with high accuracy. Moreover, the results obtained by Power series are in close agreement with the results obtained by Chebyshev polynomials. Also, the Integrated Standard Collocation Method produced better results than the Perturbed Integrated Collocation method and the methods yield the desired accuracy when the results are compared with the exact solutions.

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