MODELING AND ANALYTICAL SIMULATION OF MICROBIAL FATE AND TRANSPORT PHENOMENA IN POROUS MEDIA

I. B. S. MOHAMMED AND R. O. OLAYIWOLA
Department of Mathematics,
Federal University of Technology, Minna, Nigeria.
E-mail: olayiwola.rasaq@futminna.edu.ng babashabafu@gmail.com
Phone No: +234-805-254-8167

Abstract

Concern about pathogen contamination of groundwater and the use of microbial agents in the cleanup of groundwater has highlighted the need for an improved understanding of the fate and transport of microbes in the subsurface. This paper presents an analytical method to describe the physical, chemical and biological processes governing the simultaneous transport of microbes and nutrient in porous media. The governing equations account for the net flux of microbes by convection and dispersion, the decay and growth rates of microbes, the chemotaxis/chemotactic and the deposition of microbes on solid matrix. The decay of microbes is assumed to be a first-order reaction and the growth of microbes is assumed to follow the Monod equation. The existence and uniqueness of solution was examined. The coupled non-linear partial differential equations describing the phenomenon have been decoupled using parameter-expanding method and solved analytically using eigenfunction expansion technique. It is clear from all the results obtained that chemotaxis and sedimentation play a significant role in the transport of microbial cells through porous media.

Keywords: Microbes, microbial transport, chemotaxis, sedimentation, porous media, analytical solution.

Introduction

There has been a lot of interest in the study of fate and transport of microbes through porous media. The study is of practical nature since viruses and bacteria are responsible for some of the deadliest diseases in history, such as AIDS, the plague and flu, and yet bacteria perform the most important roles in maintaining life on this planet. Bacteria are the planet's recyclers, plant nurturer and undertakers (Christner, Morris, Foreman, Cai, Sands, 2008).

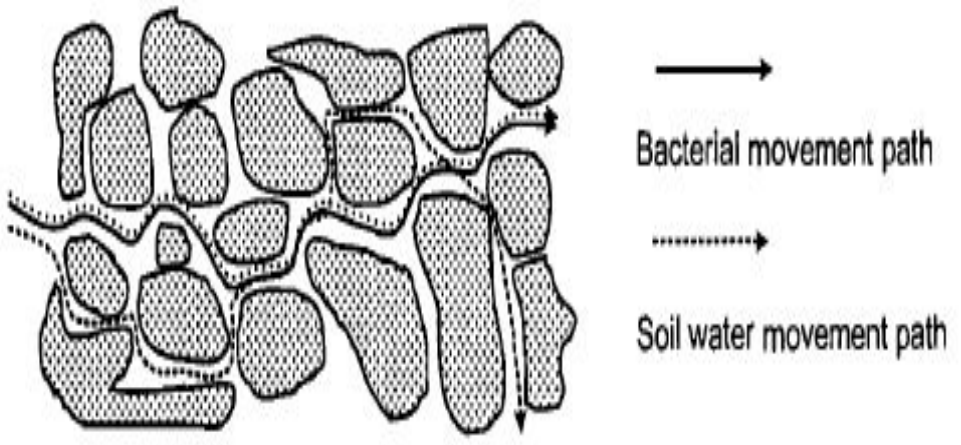
Corapcioglu and Haridas (1984) developed a model for both virus and bacteria considering the environments factors such as rainfall, soil moisture, temperature, oxygen, nutrients, etc. They found that these factors affect microbial transport. Ginn et al, (2002) in the review of physical, chemical and biological processes governing microbial transport in the saturated subsurface introduced novel conceptual models of the interactions between cell surface structures and other surfaces.

Tufenkji (2007) reviewed critically traditional approaches used to model microbial transport and fate in saturated porous media. Sen, Das, Khilar, and Suraishkumar, (2005) presented a comprehensive mathematical model for microbial transport and fate coupling with both physicochemical and biological phenomena as well as incorporation of chemotaxis/chemotactic in porous media but considered sedimentation of bacteria negligible. The numerical solution of the model was obtained using a fully implicit finite difference scheme.

Sedimentation is filtration due to gravity (Corapcioglu, & Haridas, 1984; McDowell-Boyer, Hunt & Sitar, 1986) and depends on particle buoyancy (Wan, Tokunaga & Tsang, 1995).

However, cultured microorganisms are typically larger and sometimes denser than their native counterparts (Harvey, 1997) and may involve sizeable buoyancy-driven filtration (Ginn, Wood, Nelson, Scheibe, Murphy & Clement, 2002). Thus the need for the present paper arises. Here, we incorporated sedimentation and obtained an analytical solution for describing the fate and transport of microbes in porous media. We determine the criteria for the existence of unique solution.

Model Formulation



Flow Diagram (Jiang, 2005)

Following sen et al. (2005), the transport of microbial in porous media is described by the following equations:

$$\varepsilon\frac{\partial C_b}{\partial t}=D_b\varepsilon\frac{\partial^2 C_b}{\partial x^2}-\varepsilon\left(v_p+\frac{(\rho_s-\rho)gd_s}{18\mu}+\frac{vs^2R_tk_d}{(k_d+C_f)^2}\frac{\partial C_f}{\partial x}\right)\frac{\partial C_b}{\partial x}+ \varepsilon\left(k_{g\max}\left(\frac{C_f}{k_s+C_f}\right)+k\right)C_b+k_1\rho_b(\sigma-\sigma_0)-k_2\varepsilon C_b \quad (1)$$

$$\frac{\partial(\rho_b\sigma)}{\partial t}=\rho_b\left(k_{g\max}\left(\frac{C_f}{k_s+C_f}\right)+k\right)\sigma-k_1\rho_b(\sigma-\sigma_0)+k_2\varepsilon C_b \quad (2)$$

The transport of nutrient is described by the following equation:

$$\varepsilon\frac{\partial C_f}{\partial t}+\frac{\partial(\rho_s k_f C_f)}{\partial t}=D_f\varepsilon\frac{\partial^2 C_f}{\partial x^2}-v_p\varepsilon\frac{\partial C_f}{\partial x}-\varepsilon\frac{k_{g\max}}{Y}\left(\frac{C_f}{k_s+C_f}\right)C_b- \rho_b\frac{k_{g\max}}{Y}\left(\frac{C_f}{k_s+C_f}\right)\sigma \quad (3)$$

For a packed column of length L, the initial and boundary conditions can be written as:

$$\left.\begin{array}{lll} C_b(x,0)=0, & C_b(0,t)=C_{b0}, & \left.\dfrac{\partial C_b}{\partial x}\right|_{x=L}=0 \\ & \sigma(x,0)=0 & \\ C_f(x,0)=0, & C_f(0,t)=C_{f0}, & \left.\dfrac{\partial C_f}{\partial x}\right|_{x=L}=0 \end{array}\right\} \quad (4)$$

where C_b is the microbial concentration (kg/m^3), ε is the porosity of the medium, v_c is the chemotactic velocity of microbes (m/s), v_p is the pore water velocity (m/s), v_g is the sedimentation velocity of microbes (m/s), D_b is the dispersion coefficient for microbes (m^2/s), r_r is the rate of release of captured or deposited microbes ($kg/m^3 s$), r_c is the rate of capture of microbes (freely suspended microbes) ($kg/m^3 s$), r_{gf} is the growth rate of microbes (freely suspended microbes) ($kg/m^3 s$), r_{df} is the decay rate of microbes (freely suspended microbes) ($kg/m^3 s$), σ is the volume of captured or deposited microbes per unit volume of porous medium (m^3/m^3), r_{gs} is the growth rate of capture or deposited microbes ($kg/m^3 s$), r_{ds} is the decay rate of capture or deposited microbes ($kg/m^3 s$), ρ_b is the density of microbes (kg/m^3), C_f is the nutrient (substrate) concentration (kg/m^3), S_f is the mass of adsorbed nutrient per unit mass of solid matrix (kg/kg), D_f is the dispersion coefficient for nutrient (m^2/s) and ρ_s is the bulk density of dry solid matrix (kg/m^3), s is the one-dimensional cell swimming speed (m/s), R_t is the number of receptors on the microbial cell surface, k_d is the dissociation constant for the receptor-attractant complex and v is the differential tumbling frequency which represents the fractional change in cell run time per unit temporal change in receptor occupancy, v_g is the sedimentation velocity (acting vertically downward), ρ_s is the cell density, ρ is the solution density, g is the gravitational acceleration, μ is the dynamics viscosity and d_s is the cell diameter (treated as a sphere), k_1 is the release rate coefficient for captured cells (s^{-1}), k_2 is the capture rate coefficient for free cells and σ_0 is minimum captured cell concentration, k_{gf} and k_{gs} are specific growth rates for free and captured cells (s^{-1}), respectively, $k_{g\max}$ is the maximum specific growth rate (s^{-1}) and k_s is Monod constant for the essential nutrient (kg/m^3), k_{df} and k_{ds} are specific decay rates for free and captured cells (s^{-1}), Y is the yield coefficient, k_f is the partition coefficient (m^3/kg).

Method of Solution

Non-dimensionalization: Here, we non-dimensionalize equations (1) – (4), using the following dimensionless variables:

$$t'=\frac{v_p t}{L}, \qquad x'=\frac{x}{L}, \qquad C=\frac{C_b}{C_{b0}}, \qquad \theta=\frac{\sigma}{\sigma_0}, \qquad \phi=\frac{C_f}{C_{f0}} \quad (5)$$

and obtain

$$\frac{\partial C}{\partial t} = D_1 \frac{\partial^2 C}{\partial x^2} - \left(1 + \alpha_2 + \frac{\alpha}{(a+\phi)^2}\frac{\partial \phi}{\partial x}\right)\frac{\partial C}{\partial x} + \left(\gamma\left(\frac{\phi}{b+\phi}\right) + \delta\right)C + \beta(\theta - 1) - \lambda C \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \left(\gamma\left(\frac{\phi}{b+\phi}\right) + \delta\right)\theta - \beta_1(\theta - 1) + \lambda_1 C \tag{7}$$

$$\frac{\partial \phi}{\partial t} + \alpha_1 \frac{\partial \phi}{\partial t} = D_2 \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial \phi}{\partial x} - \gamma_1\left(\frac{\phi}{b+\phi}\right)C - \gamma_2\left(\frac{\phi}{b+\phi}\right)\theta \tag{8}$$

together with initial and boundary conditions:

$$\left.\begin{array}{lll} C(x,0) = 0, & C(0,t) = 1, & \left.\dfrac{\partial C}{\partial x}\right|_{x=1} = 0 \\ & \theta(x,0) = 0 & \\ \phi(x,0) = 0, & \phi(0,t) = 1, & \left.\dfrac{\partial \phi}{\partial x}\right|_{x=1} = 0 \end{array}\right\}, \tag{9}$$

where

$$D_1 = \frac{D_b}{Lv_p}, \quad \alpha = \frac{vs^2 R_t k_d}{Lv_p C_{f0}}, \quad a = \frac{k_d}{C_{b0}}, \quad b = \frac{k_s}{C_{f0}}, \quad \gamma = \frac{Lk_{g\max}}{v_p},$$

$$\delta = \frac{kL}{v_p}, \quad \beta = \frac{Lk_1\rho_b\sigma_0}{\varepsilon C_{b0} v_p}, \quad \lambda = \frac{k_2 L}{v_p}, \quad \lambda_1 = \frac{Lk_2\varepsilon C_{b0}}{\rho_b\sigma_0 v_p}, \quad \beta_1 = \frac{k_1 L}{v_p},$$

$$\alpha_1 = \frac{Lk_f\rho_s}{\varepsilon}, \quad D_2 = \frac{D_f}{Lv_p}, \quad \gamma_1 = \frac{Lk_{g\max}C_{b0}}{YC_{f0}v_p}, \quad \gamma_2 = \frac{Lk_{g\max}\rho_b\sigma_0}{Y\varepsilon C_{f0}v_p}, \quad \alpha_2 = \frac{(\rho_s - \rho)gd_s}{18\mu v_p}$$

Existence and Uniqueness of Solution

Theorem 1: Let $D_1 = D_2 + D, \quad \alpha = \alpha_1 = \alpha_2 = \gamma = \gamma_1 = \gamma_2 = 0, \quad \lambda = \lambda_1 = \beta_1 = \delta$. Then the equations (6) – (8) with initial and boundary conditions (9) has a unique solution for all $t \geq 0$ ·

Proof: Let $D_1 = D_2 + D, \quad \alpha = \alpha_1 = \alpha_2 = \gamma = \gamma_1 = \gamma_2 = 0, \quad \lambda = \lambda_1 = \beta_1 = \delta$ and $\psi(x,t) = C(x,t) + \phi(x,t)$, we obtain

$$\frac{\partial \theta}{\partial t} = \delta(1 + C), \qquad \theta(x,0) = 0 \tag{10}$$

$$\frac{\partial \psi}{\partial t} = D\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial \psi}{\partial x} + \beta(\theta - 1), \quad \psi(x,0) = 0, \quad \psi(0,t) = 2, \quad \psi_x(1,t) = 0 \tag{11}$$

Using direct integration and eigenfunction expansion method, we obtain the solution of problem (10) as

$$\theta(x,t) = \delta(t + k(x,t)) \tag{12}$$

where

$$k(x,t) = \int C(x,t)dt$$

and the solution of problem (11) as

$$\psi(x,t) = 2 + \beta\sum_{n=1}^{\infty} V_n(t)\sin\left(\frac{2n-1}{2}\right)\pi x \tag{13}$$

where

$$V_n(t)=\int_0^t \exp\left(-D\left(\frac{(2n-1)\pi}{2}\right)^2(t-\tau)\right)F_n(\tau)d\tau$$

$$F_n(t)=2\delta T_n(t)+\frac{4}{(2n-1)\pi}(\delta t-1)$$

$$T_n(t)=\int_0^1 k(x,t)\sin\left(\frac{2n-1}{2}\right)\pi x dx$$

Then, we obtain

$$C(x,t)=\left(2+\beta\sum_{n=1}^{\infty}V_n(t)\sin\left(\frac{2n-1}{2}\right)\pi x\right)-\phi(x,t) \quad (14)$$

$$\phi(x,t)=\left(2+\beta\sum_{n=1}^{\infty}V_n(t)\sin\left(\frac{2n-1}{2}\right)\pi x\right)-C(x,t) \quad (15)$$

Hence, there exists a unique solution of problem (21) – (23). This completes the proof.

Analytical Solution

We let $m=1+\alpha_2$ in (6) and solve equations (6) – (9) using parameter-expanding method (where details can be found in [5]) and eigenfunctions expansion method (where details can be found in [8]).

We rewrite equations (6) - (8) in the form:

$$\frac{\partial C}{\partial t}=D_1\frac{\partial^2 C}{\partial x^2}-\left(m+\frac{\alpha}{(a+\phi)^2}\frac{\partial\phi}{\partial x}\right)\frac{\partial C}{\partial x}+\left(\gamma\left(\frac{\phi}{b+\phi}\right)+\delta\right)C+\beta(\theta-1)-\lambda C \quad (16)$$

$$\frac{\partial\theta}{\partial t}=\left(\gamma\left(\frac{\phi}{b+\phi}\right)+\delta\right)\theta-\beta_1(\theta-1)+\lambda_1 C \quad (17)$$

$$\frac{\partial\phi}{\partial t}+\alpha_1\frac{\partial\phi}{\partial t}=D_2\frac{\partial^2\phi}{\partial x^2}-f\alpha\frac{\partial\phi}{\partial x}-\gamma_1\left(\frac{\phi}{b+\phi}\right)C-\gamma_2\left(\frac{\phi}{b+\phi}\right)\theta\,, \quad (18)$$

where $f\alpha=1$

We let

$m=h\alpha,\qquad \gamma=p\alpha,\qquad \lambda_1=q\alpha,\qquad \gamma_1=r\alpha,\qquad \gamma_2=s\alpha$

Suppose that the solution of equations (16) – (18) can be expressed as:

$$\left.\begin{aligned}C(x,t)&=C_0(x,t)+\alpha C_1(x,t)+\ldots\\ \theta(x,t)&=\theta_0(x,t)+\alpha\theta_1(x,t)+\ldots\\ \phi(x,t)&=\phi_0(x,t)+\alpha\phi_1(x,t)+\ldots\end{aligned}\right\} \quad (19)$$

Substituting (19) into (16) – (18) and processing, we obtain

$$\frac{\partial C_0}{\partial t}=D_1\frac{\partial^2 C_0}{\partial x^2}+(\delta-\lambda)C_0+\beta(\theta_0-1) \quad (20)$$

$$C_0(x,0)=0,\qquad C_0(0,t)=1,\qquad \left.\frac{\partial C_0}{\partial x}\right|_{x=1}=0$$

$$\frac{\partial\theta_0}{\partial t}=(\delta-\beta_1)\theta_0+\beta_1 \quad (21)$$

$$\theta_0(x,0)=0$$

$$\frac{\partial\phi_0}{\partial t}+\alpha_1\frac{\partial\phi_0}{\partial t}=D_2\frac{\partial^2\phi_0}{\partial x^2} \quad (22)$$

$$\phi_0(x,0)=0, \qquad \phi_0(0,t)=1, \qquad \left.\frac{\partial \phi_0}{\partial x}\right|_{x=1}=0$$

$$\frac{\partial C_1}{\partial t}=D_1\frac{\partial^2 C_1}{\partial x^2}+(\delta-\lambda)C_1-h\frac{\partial C_0}{\partial x}-\frac{1}{(a+\phi_0)^2}\frac{\partial \phi_0}{\partial x}\frac{\partial C_0}{\partial x}+p\left(\frac{\phi_0}{b+\phi_0}\right)C_0+\beta\theta_1 \tag{23}$$

$$C_1(x,0)=0, \qquad C_1(0,t)=0, \qquad \left.\frac{\partial C_1}{\partial x}\right|_{x=1}=0$$

$$\frac{\partial \theta_1}{\partial t}=(\delta-\beta_1)\theta_1+p\left(\frac{\phi_0}{b+\phi_0}\right)\theta_0+qC_0 \tag{24}$$

$$\theta_1(x,0)=0$$

$$\frac{\partial \phi_1}{\partial t}+\alpha_1\frac{\partial \phi_1}{\partial t}=D_2\frac{\partial^2 \phi_1}{\partial x^2}-f\frac{\partial \phi_0}{\partial x}-r\left(\frac{\phi_0}{b+\phi_0}\right)C_0-s\left(\frac{\phi_0}{b+\phi_0}\right)\theta_0 \tag{25}$$

$$\phi_1(x,0)=0, \qquad \phi_1(0,t)=0, \qquad \left.\frac{\partial \phi_1}{\partial x}\right|_{x=1}=0$$

Using integrating factor method and eigenfunctions expansion method, we obtain the solution of equations (20) - (25) as

$$C_0(x,t)=1+\sum_{n=1}^{\infty}\left(q_1\left(e^{Bt}-e^{p_1t}\right)-q_2\left(1-e^{p_1t}\right)\right)\sin\left(\frac{2n-1}{2}\right)\pi x \tag{26}$$

$$\theta_0(x,t)=A\left(1-e^{Bt}\right) \tag{27}$$

$$\phi_0(x,t)=1 \tag{28}$$

$$C_1(x,t)=\sum_{n=1}^{\infty}\left(\begin{array}{l}\beta\sum_{n=1}^{\infty}\left(q_3\left(e^{Bt}-e^{p_1t}\right)-q_4\left(e^{-Bt}-e^{p_1t}\right)-q_5\left(1-e^{p_1t}\right)-m_4te^{p_1t}\right)+\\ m_8\sum_{n=1}^{\infty}\left(q_6\left(e^{Bt}-e^{p_1t}\right)+q_7\left(1-e^{p_1t}\right)-(q_2+q_1)te^{p_1t}\right)+\\ \frac{4\beta}{(2n-1)\pi}\left(-q_8\left(1-e^{p_1t}\right)+q_9\left(e^{-Bt}-e^{p_1t}\right)-q_{10}\left(e^{Bt}-e^{p_1t}\right)\right)-\\ h\sum_{n=1}^{\infty}\frac{\left(1+(-1)^{2n}\right)}{2}\left(\begin{array}{l}q_6\left(e^{Bt}-e^{p_1t}\right)+q_7\left(1-e^{p_1t}\right)+\\ (q_2-q_1)te^{p_1t}\end{array}\right)\end{array}\right)\sin\left(\frac{2n-1}{2}\right)\pi x \tag{29}$$

$$\theta_1(x,t)=\sum_{n=1}^{\infty}\left(m_2e^{Bt}+m_3e^{-Bt}-m_4e^{p_1t}+m_5\right)\sin\left(\frac{2n-1}{2}\right)\pi x+m_6\left(1-e^{-Bt}\right)+m_7\left(e^{Bt}-e^{-Bt}\right) \tag{30}$$

$$\phi_1(x,t)=\sum_{n=1}^{\infty}\left(\begin{array}{l}(p_3+p_4)\left(1-e^{p_2t}\right)+p_5\left(e^{Bt}-e^{p_2t}\right)-\\ \sum_{n=1}^{\infty}\left(p_6\left(e^{Bt}-e^{p_2t}\right)+p_7\left(e^{p_1t}-e^{p_2t}\right)+p_8\left(1-e^{p_2t}\right)\right)\end{array}\right)\sin\left(\frac{2n-1}{2}\right)\pi x \tag{31}$$

where

$$A=\frac{\beta_1}{\beta_1-\delta},\quad B=\beta_1-\delta,\quad B_1=\delta-\lambda,\quad p_1=\left(B_1-D_1\left(\left(\frac{2n-1}{2}\right)\pi\right)^2\right),\quad q_1=\frac{4\beta A}{(2n-1)p_1\pi},$$

$$q_2=\frac{4(\beta(A-1)+B_1)}{(2n-1)p_1\pi},\quad D_3=\frac{D_2}{1+\alpha_1},\quad m=\frac{pA}{b+1}+q,\quad m_1=\frac{pA}{b+1},\quad m_2=\frac{qq_1}{2B},\quad m_5=\frac{qq_2}{B},$$

$$m_3=\frac{qq_1}{B+p_1}-\frac{qq_1}{2B}+\frac{qq_2}{B+p_1}-\frac{qq_2}{B},\quad m_4=\frac{q(q_1+q_2)}{B+p_1},\quad m_6=\frac{m}{B},\quad m_7=\frac{m_1}{2B},\quad m_8=\frac{p}{b+1},$$

$$q_3=\frac{m_2}{B-p_1},\quad q_4=\frac{m_3}{B+p_1},\quad q_5=\frac{m_5}{p_1},\quad q_6=\frac{q_1}{B-p_1},\quad q_7=\frac{q_2}{p_1},\quad q_8=\frac{m_6}{p_1},\quad q_9=\frac{m_6-m_7}{B+p_1},$$

$$q_{10}=\frac{m_7}{B-p_1},\qquad q_{12}=\frac{r}{(1+\alpha_1)(b+1)},\qquad q_{13}=\frac{sA}{(1+\alpha_1)(b+1)},\qquad p_2=-D_3\left(\left(\frac{2n-1}{2}\right)\pi\right)^2,$$

$$p_3=\frac{4q_{12}}{(2n-1)p_2\pi},\qquad p_4=\frac{4q_{13}}{(2n-1)p_2\pi},\qquad p_5=\frac{4q_{13}}{(2n-1)(B-p_2)\pi},\qquad p_6=\frac{q_1q_{12}}{B-p_2},$$

$$p_7=\frac{q_{12}(q_2-q_1)}{p_1-p_2},\quad p_8=\frac{q_2q_{12}}{p_2}$$

The computations were done using computer symbolic algebraic package MAPLE.

Results and Discussion

The systems of partial differential equations describing the physical, chemical and biological processes governing the simultaneous transport of microbes and nutrient in the presence of filtration due to gravity are solved analytically using parameter-expanding method and eigenfunction expansion technique. Analytical solutions of equations (16) - (19) are computed for the values of $\alpha=1,\ \alpha_2=0.4,\ D_1=0.4,\ D_2=0.3,\ \beta=0.2,\ \beta_1=0.2,$ $\lambda=0.5,\ \lambda_1=0.5,\ \delta=1,\ \alpha_1=1,\ a=1,\ b=20,\ \gamma=1,\ \gamma_1=1,\ \gamma_2=1$

The following figures explain the distribution of volume of captured microbes and microbial and nutrient concentration against different dimensionless parameters.

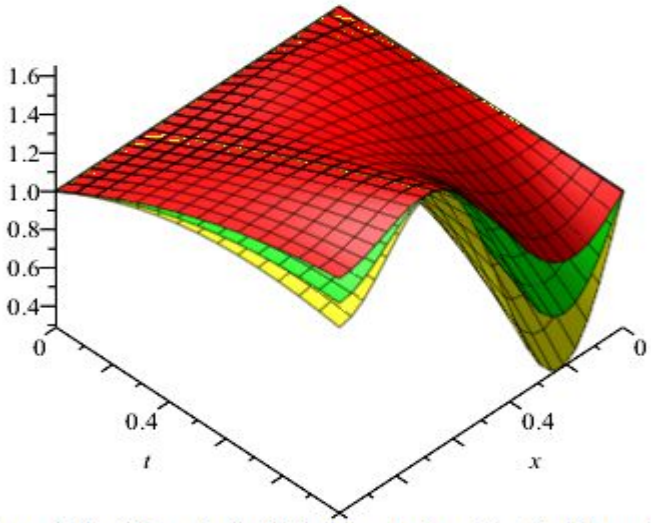


Figure 1: Variation of microbial concentration $C(x,t)$ with gravity number α_2

From Figure 1, we can conclude that with the increase of Gravity number (α_2), microbial concentration decreases along the temporal and spatial directions.

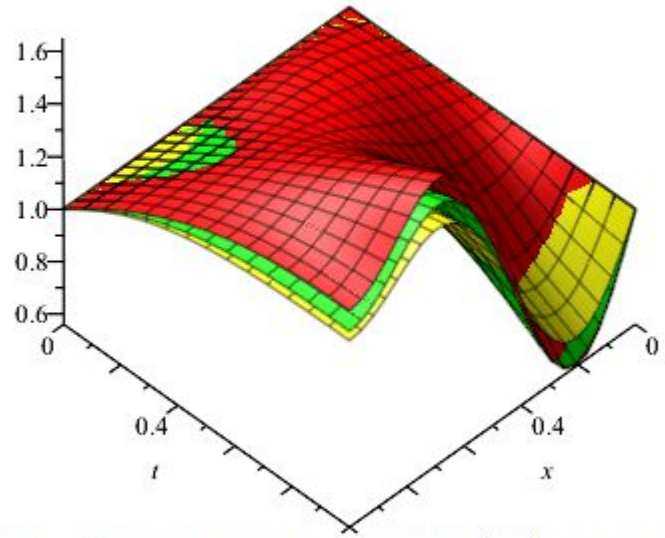


Figure 2: Variation of microbial concentration $C(x, t)$ with dispersion coefficient for microbes D_1

From Figure 2, we can conclude that with the increase of dispersion coefficient for microbes (D_1), microbial concentration decreases along the temporal and spatial directions.

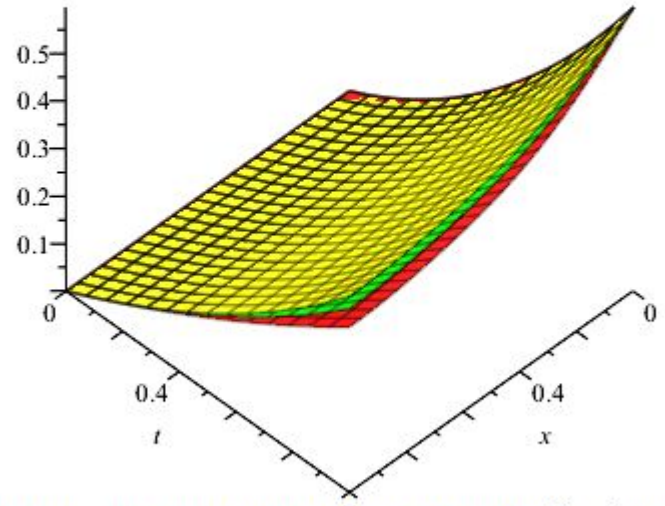


Figure 3: Variation of volume of captured microbes $\theta(x, t)$ with dispersion coefficient for microbes D_1

From Figure 3, we can conclude that with the increase of dispersion coefficient for microbes (D_1), the volume of captured microbes increases along the temporal and spatial directions.

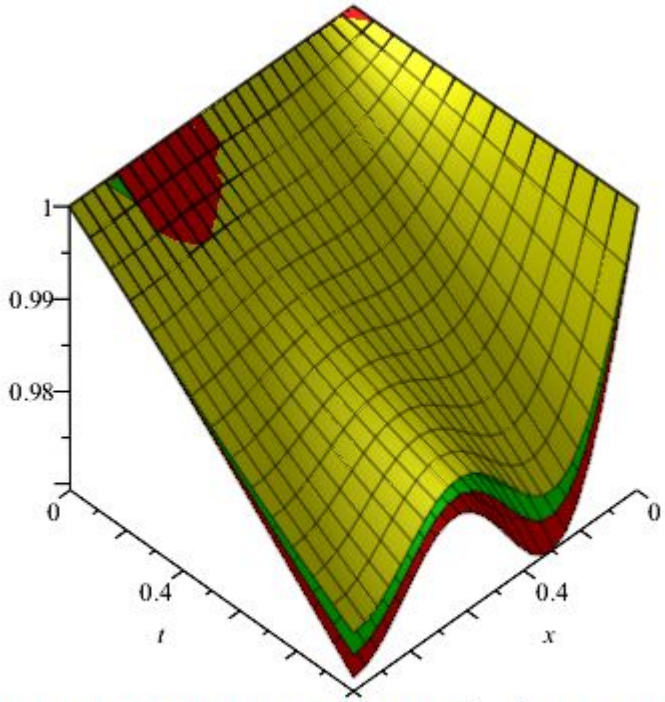


Figure 4: Variation of nutrient concentration $\phi(x, t)$ with dispersion coefficient for nutrient D_2

From Figure 4, we can conclude that with the increase of dispersion coefficient for nutrient (D_2), nutrient concentration increases along the temporal and spatial directions.

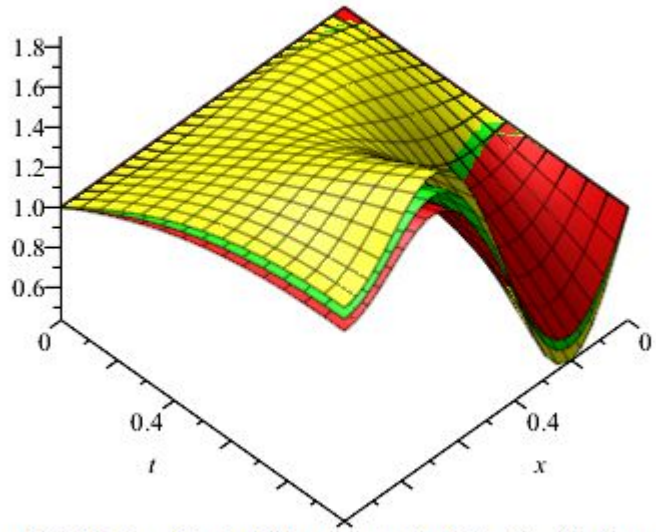


Figure 5: Variation of microbial concentration $C(x, t)$ with release rate coefficient for captured cells β

From Figure 5, we can conclude that with the increase of release rate coefficient for captured cells(β), microbial concentration increases along the temporal and spatial directions.

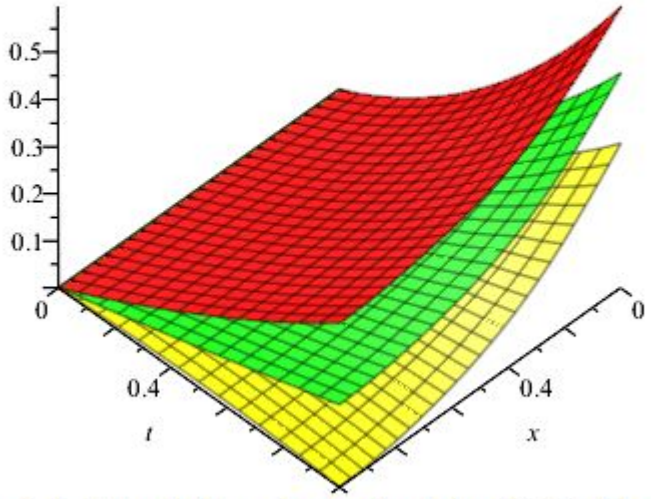


Figure 6: Variation of volume of captured microbes $\theta(x, t)$ with release rate coefficient for captured cells β

From Figure 6, we can conclude that with the increase of release rate coefficient for captured cells(β), the volume of captured microbes decreases along the temporal and spatial directions.

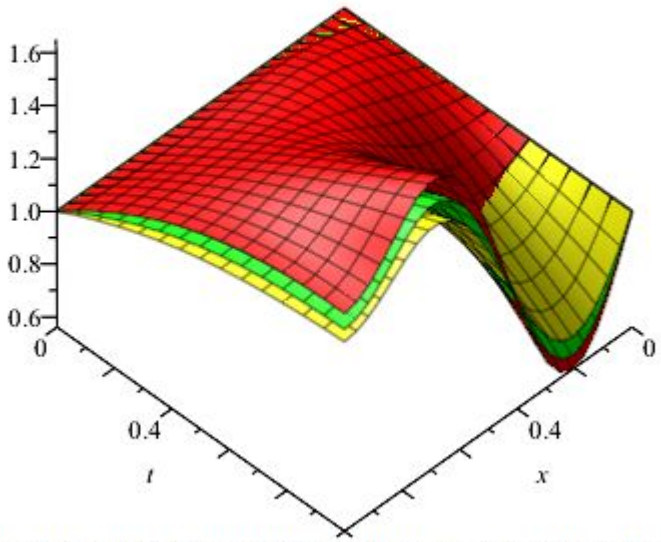


Figure 7: Variation of microbial concentration $C(x, t)$ with capture rate coefficient for free cells λ

From Figure 7, we can conclude that with the increase of capture rate coefficient for free cells (λ), microbial concentration decreases along the temporal and spatial directions.

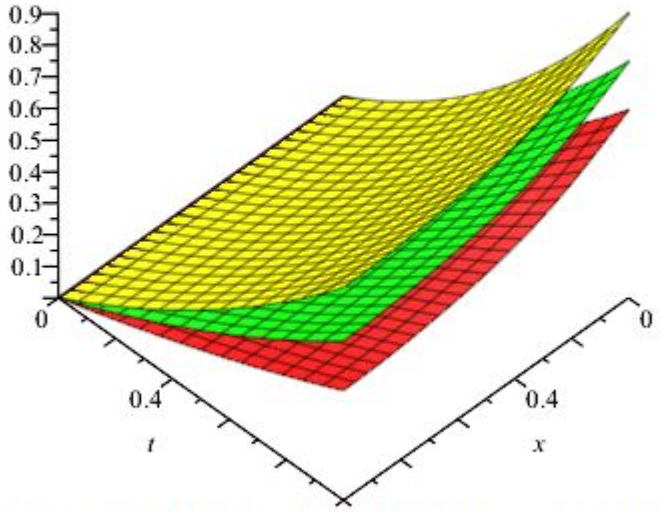


Figure 8: Variation of volume of captured microbes $\theta(x, t)$ with capture rate coefficient for free cells λ

From Figure 8, we can conclude that with the increase of capture rate coefficient for free cells (λ), volume of captured microbes increases along the temporal and spatial directions.

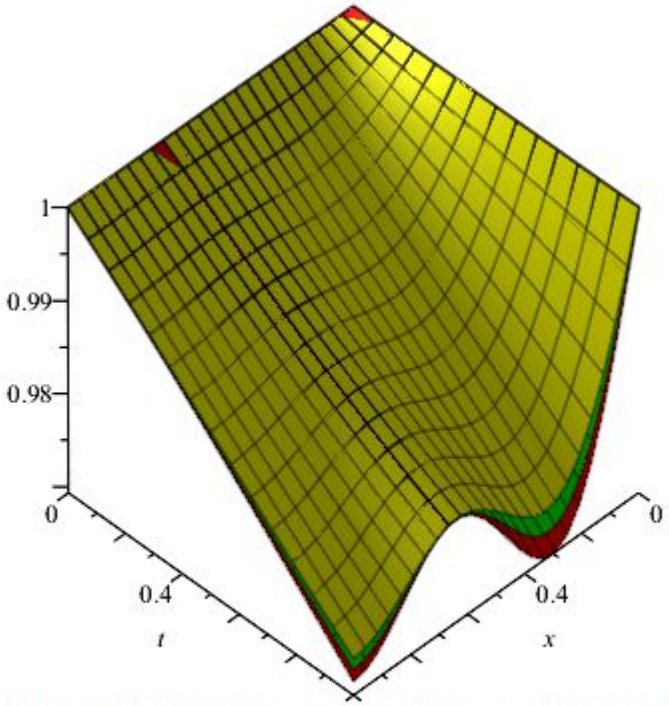


Figure 9: Variation of nutrient concentration $\phi(x, t)$ with capture rate coefficient for free cells λ

From Figure 9, we can conclude that with the increase of capture rate coefficient for free cells (λ), nutrient concentration increases along the temporal and spatial directions.

Conclusion

In this work, we studied the physical, chemical and biological processes governing the simultaneous transport of microbes and nutrients in porous media. The model used allows some essential insight of how chemotaxis and sedimentation can change the concentration of free and captured microbes. Based on our results, we state that:

(i) Gravity number decreases the microbial concentration.
(ii) Dispersion coefficient for microbes enhances the volume of captured microbes and decreases the microbial concentration.
(iii) Dispersion coefficient for nutrient enhances the nutrient concentration.
(iv) Release rate coefficient for captured cells enhances the microbial concentration and decreases the volume of captured microbes.
(v) Capture rate coefficient for free cells decreases the microbial concentration and enhances the volume of captured microbes and nutrient concentration.

The main conclusion is that increase in weight or size of free microbes change significantly the concentration of free microbes, possibly many of them felled and were trapped in the pore matrix due to weight and size. This has negative implication on their survival when moving with soil water through the pore. Thus, it is crucial to prevent microbes from adding weight or size as it may affect their movement through porous media.

References

Christner, B. C., Morris, C. E., Foreman, C. M., Cai, R., & Sands, D. C. (2008). Ubiquity of biological ice nucleators in snowfall. *Science,* 319, (5867), 12-14. doi:10.1126/science.1149757.

Corapcioglu, M. Y. & Haridas, A. (1984). Transport and fate of microorganisms in porous media: A vestigation. *Journal of Hydrology,* 72, 149 - 169.

Ginn, T. R., Wood, B. D., Nelson, K. E., Scheibe, T. D., Murphy, E. M. & Clement, T. P. (2002). Processes in microbial transport in the natural subsurface. *Advances in Water Resources*, 25, 1017 – 1042.

Harvey, R. W. (1997). Microorganisms as tracers in groundwater injection and recovery experiments: A review. *FEMSMicrobiol Rev.*, 20, 461 – 472.

He, J. H. (2006). Some asymptotic methods for strongly nonlinear equations, *Int. J. Modern Phys. B.* 20(10), 1141 – 1199

Jiang, G. (2005). *Bacterial transport and deposition in an intact soil lysimeter and packed sand columns.* Master of Applied Science Thesis, Lincoln University New Zealand 32-33.

McDowell-Boyer, L. M., Hunt, J. R. & Sitar, N. (1986). Particle transport through porous media. *Water Resour Res.* 31, 1627 – 1636.

Myint-U, T. & Debnath, L. (1987). *Partial differential equation for scientists and engineers.* New Jersey: PTR Prentice – Hall, Englewood Cliffs.

Sen, T. K., Das, D., Khilar, K. C. & Suraishkumar, G. K. (2005). Bacterial transport in porous media: New aspects of the Mathematical model. *Colloids and Surfaces A: Physicochemical and Engineering Aspect,* 260 (1 - 3), 53 – 62.

Tufenkji, N. (2007). Modeling microbial transport in porous media: Traditional approaches and recent developments. *Advances in Water Resources*, 30, 1455 – 1469.

Wan, J., Tokunaga, T. K. & Tsang, C. F. (1995). Bacterial sedimentation through a porous medium. *Water Resour Res.,* 22, 1901 – 1921.