MODELING AND ANALYTICAL SIMULATION OF TRANSIENT FREE CONVECTION MAGNETOHYDRODYNAMICS FLOW BETWEEN TWO HEATED VERTICAL PLATES

OLADEJI, S. O., JIYA, M., & OLAYIWOLA, R. O.
Department of Mathematics, Federal University of Technology, Minna, Nigeria
E-mail: olatundesolomon486@gmail.com, olayiwola.rasaq@futminna.edu.ng
Phone No: +234-806-774-3443

Abstract
This paper presents an analytical method for describing heat dissipation and magnetic field effects on unsteady free convection magnetohydrodynamics flow between two heated vertical plates. It is assumed that the initial velocity increases lengthwise. The equations are solved using parameter-expanding method and Eigenfunctions expansion technique. The results obtained shows that the Prandth number and Reynolds number have significant effects on the velocity and temperature distribution.

Keywords: Eigenfunctions expansion technique, Free convection flow, Magnetic field, Magnetohydrodynamics, Parameter-expanding method

Introduction
A magnetic field is the magnetic influence of electric currents and magnetic materials. The magnetic field at any given point is specified by both a direction and a magnitude (or strength). As such it is a vector field (Oladeji, 2015). In everyday life, magnetic fields are most often encountered as an invisible force created by permanent magnets which pull on ferromagnetic materials such as iron, cobalt or metal and attract or repel other magnets. Magnetic fields are very widely used throughout modern technology, particularly in electrical engineering and electro mechanics. The earth produces its own magnetic fields, which is important in navigation. Rotating magnetic fields are used in both electric motors and generators. Magnetic forces give information about the charge carriers in a material through the Hall Effect. The interaction of magnetic fields in electric devices such as transformers is studied in the discipline of magnetic circuits (Oladeji, 2015).

Magneto hydrodynamics is the state of the dynamics of electrically conducting fluids. Examples of such fluids include plasma, liquid metal and saltwater or electrolytes. The fundamental concept behind MHD (Magneto hydro dynamics) is that magnetic fields can induce currents in a moving conductive fluid, which in turn, creates forces on the fluid, and also changes the magnetic field itself. The set of equations which describe MHD are a combination of the Nervier-Stokes equation of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations have to be solved simultaneously either analytically or numerically (Olajuwon & Dahimire,2013).

Unsteady free convection flows in a porous medium have received much attention in recent time due to its wide applications in geothermal and oil reservoir engineering as well as other geophysical and astrophysical studies. Bhaskar (2014) investigated the effects of mass transfer on unsteady free convection MHD flow between two heated vertical plates in the presence of transverse magnetic field. Lakshmi *et al.* (2013) studied the effects of Heat Generation and Viscous Dissipation on MHD Heat and Mass Diffusion Flow Past a Surface. Their results show that the Heat Generation and Viscous Dissipation have significance influence on the flow, heat and mass transfer.

Olajuwon and Dahimire (2013) studied the effects of thermo-diffusion and thermal radiation on unsteady heat and mass transfer. The results show that the observed parameters have significance influence on the flow, heat and mass transfer. Chand, Kumar and Sharma (2012) investigated the hydro magnetic oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and Soret effect. When one plate of the channel is kept stationery and the other is moving with uniform velocity. And the result showed that the Lorentz force parameter i.e. the Hartmann number contributes to reduce the velocity and the skin friction profile.

Ibrahim (2014) examined the unsteady MHD convection heat and mass transfer past an infinite vertical plate embedded in a porous medium with radiation and chemical reaction under the influence of dufour and soret effects. Ashkkunar *et al.* (2013) studied closed form solutions of heat and mass transfer in the flow of a MHD viscous-elastic fluid over a porous stretching sheet. It is found that the heat and mass transfer distribution decreases with the increasing values of the viscous-elastic parameter.

The objective of this paper is to obtain an analytical solution for describing the heat dissipation and magnetic field effects on unsteady free convection magneto hydrodynamics flow between two heated vertical plates.

Materials and Methods

The physical configuration illustrating the problem under consideration is as shown in figure 1.

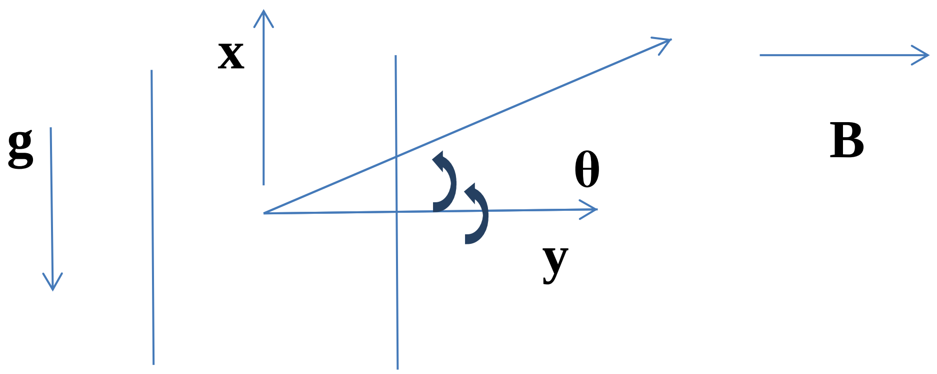

Figure 1: Geometry of the flow field

The vertical plates are fixed and the fluid flows between the two plates pointing to vertical upward direction. The $x-axis$, taken as the axis of the channel, is pointing to vertical upward direction through the idle of the two plates. The $y-axis$ is along the horizontal direction. The plates of the channelled are kept at $y=\pm 1$. g acts in the vertical downward direction while B_0 acts at an angle $0 \le \theta \ge \frac{\pi}{2}$ to the $y-axis$. In order to derive the fundamental equation, we assume that

1. The fluid is Newtonian, viscous, incompressible
2. The Hall affect, electrical effect and polarization effect, are neglected
3. The variation of expansion coefficient respect to temperature difference are considered
4. The boundary layer is assumed to be thin relative to the distance between the two plates
5. The pressure gradient across the boundary layer is neglected.
6. The induced magnetic field is assumed to be very small
7. The viscous dissipation is considered

8. One of the plates is assume adiabatic
9. It is also assume that initial velocity depends on space variables (i.e., initial velocity increases lengthwise).

Considering the above assumption the equations describing the unsteady free convection flow of an incompressible viscous fluid through a vertical channel are the:

Continuity equation

$$\frac{\partial u}{\partial t} = 0 \tag{1}$$

Momentum equation

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_0) - \frac{\sigma B^2 \upsilon \cos\theta u}{\rho} \tag{2}$$

Energy equation

$$\frac{\partial T}{\partial t} = \frac{k}{\rho cp}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho cp}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The initial and boundary conditions are formulated as:

$$\left.\begin{array}{lll} u(y,0) = \dfrac{\rho g L^2}{\upsilon}(T_w - T_0)\left(1 - \dfrac{y}{L}\right), & u(0,t) = 0, & u(L,t) = 0 \\ T(y,0) = T_0, \quad T(0,t) = T_0 + (T_w - T_0)(1 - e^{nt}), & & \left.\dfrac{\partial T}{\partial y}\right|_{y=1} = 0 \end{array}\right\}, \tag{4}$$

where T_w and T_0 are the temperature at the plates $y = 0$ and $y = 1$ respectively, $(n > 0)$ a real number denotes the decay factor, β is the variation of expansion coefficient, k is the thermal conductivity with respect to temperature different, β_0 acts at an angle ϕ , u and v are the velocity components, T is the temperature of the medium.

Method of Solution

Non-dimensionalization

Here, we non-dimensionalized equations (1) – (4), using the following dimensionless variables:

$$\left.\begin{array}{lll} \theta = \dfrac{T - T_0}{T_w - T_0}, & y' = \dfrac{y}{L}, & t' = \dfrac{Ut}{L} \\ P_r = \dfrac{\mu C_p}{K}, & u' = \dfrac{\upsilon u}{\beta g L^2 (T_w - T_0)}, & n' = \dfrac{Ln}{U} \end{array}\right\} \tag{5}$$

and we obtain

$$\frac{\partial u}{\partial t} = \frac{1}{R_e}\frac{\partial^2 u}{\partial y^2} + \frac{1}{R_e}\theta - Mu \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_e}\frac{\partial^2 \theta}{\partial y^2} + G\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

with the initial and boundary conditions:

$$\left.\begin{array}{ll} u(y,0)=(1-y), \quad u(0,t)=0, \quad u(1,t)=0 & 0\le y\le 1 \\ \theta(y,0)=0, \quad \theta(0,t)=\left(1-e^{nt}\right), & \left.\dfrac{\partial\theta}{\partial y}\right|_{y=1}=0 \end{array}\right\}, \qquad (8)$$

where $M=\dfrac{\sigma LB_0^2\cos^2\phi}{\rho}$ = Magnetic number, $R_e=\dfrac{LU}{\upsilon}$ = Reynolds number,

$G=\dfrac{L^3\beta^2 g^2(T_w-cT_0)}{Uc_p\upsilon}$ = Grash of number, $P_e=\dfrac{\rho c_p LU}{k}$ = Peclet number

Analytical Solution

Here, we solve equations (6) – (8) using parameter-expanding method (where details can be found in He, 2006) and eigenfunctions expansion method (where details can be found in Myint-U and Debnath, 1987).

Suppose that the solution of equations (6) – (8) can be expressed as:

$$\left.\begin{array}{l} u(x,t)=u_0(x,t)+Gu_1(x,t)+\ldots \\ \theta(x,t)=\theta_0(x,t)+G\theta_1(x,t)+\ldots \end{array}\right\} \qquad (9)$$

Substituting (9) into (6) – (8) and processing, we obtain for

G^0 :

$$\frac{\partial u_0}{\partial t}=\frac{1}{R_e}\frac{\partial^2 u_0}{\partial y^2}+\frac{1}{R_e}\theta-Mu_0,\quad u_0(y,0)=1-y,\quad u_0(0,t)=0,\quad u_0(l,t)=0 \qquad (10)$$

$$\frac{\partial\theta_o}{\partial t}=\frac{1}{p_e}\frac{\partial^2\theta_0}{\partial y^2},\quad \theta_0(y,0)=0,\quad \theta_0(0,t)=1-e^{nt},\quad \left.\frac{\partial\theta}{\partial y}\right|_{y=1}=0 \qquad (11)$$

G^1 :

$$\frac{\partial u_1}{\partial t}=\frac{1}{R_e}\frac{\partial^2 u_1}{\partial y^2}+\frac{1}{R_e}\theta_1-Mu_1,\quad u_1(y,0)=0,\quad u_1(0,t)=0,\quad u_1(1,t)=0 \qquad (12)$$

$$\frac{\partial\theta_1}{\partial t}=\frac{1}{p_e}\frac{\partial^2\theta_1}{\partial y^2}+\left(\frac{\partial u_0}{\partial y}\right)^2,\quad \theta_1(y,0)=0,\quad \theta_1(0,t)=0,\quad \left.\frac{\partial\theta}{\partial y}\right|_{y=1}=0 \qquad (13)$$

Using eigenfunctions expansion method and direct integration, we obtain the solution of equations (10) - (13) as

$$\theta_0(y,t)=1-e^{nt}+\sum_{n-1}^{\infty}\frac{4ne^{\left(-\frac{1}{p_e}\left(\frac{2n-1}{2}\right)^2\pi^2\right)t}}{(2n-1)\pi\left(n-\frac{1}{p_e}\left(\frac{2n-1}{2}\right)^2\pi^2\right)}\left(e^{\left(-\frac{1}{p_e}\left(\frac{2n-1}{2}\right)^2\pi^2\right)t}-1\right)\sin\left(\frac{2n-1}{2}\right)\pi y \qquad (14)$$

$$u(y,t)=\sum_{n=1}^{\infty}\left(\begin{array}{l}\dfrac{2}{R_e}\displaystyle\sum_{n=1}^{\infty}\frac{2(2n-1+(-1)^{2n})}{(4n-1)\pi}\left(\left(\frac{e^{-m_0t}-e^{-m_1t}}{q_0}\right)-\left(\frac{e^{-m_2t}-e^{-m_1t}}{q_1}\right)\right)\\ +\dfrac{2}{R_e}\left(\dfrac{1-(-1)^n}{n\pi}\right)\left(\dfrac{1-e^{m_1t}}{m_1}\right)-\left(\dfrac{e^{nt}-e^{m_1t}}{q_2}\right)+\dfrac{2}{n\pi}e^{-m_1t}\end{array}\right)\sin n\pi y \qquad (15)$$

$$\theta_1(y,t)=\sum_{n=1}^{\infty}\left(\frac{2(1-2n)}{(4n-1)\pi}\right)\left(\begin{array}{l}\frac{2}{R_e}\sum_{n=1}^{\infty}\frac{2(2n-1+(-1)^{2n})}{(4n-1)\pi}\left(\begin{array}{l}\frac{1}{q_2}(e^{-m_0t}-e^{-m_1t})\\+q_4(e^{-m_1t}-e^{-m_2t})-\\\frac{1}{q_8}(e^{-m_2t}-e^{-m_8t})\end{array}\right)\\+\frac{2}{R_e}\left(\frac{1-(-1)^n}{n\pi}\right)\left(\begin{array}{l}\frac{1}{q_6}(1-e^{-m_3t})+\\q_7(e^{-m_1t}-e^{-m_3t})-\frac{1}{q_8}(e^{nt}-e^{-m_8t}\end{array}\right)\\+\frac{2}{n\pi\, q_9}\left(e^{-m_1t}-e^{-m_8t}\right)\end{array}\right) \tag{16}$$

where

$m_0=\frac{2n^2\pi^2}{p_e}-\frac{2n\pi^2}{p_e}+\frac{\pi^2}{2p_e}-n$, $m_1=M+\frac{n^2\pi^2}{R_e}$, $m_2=\frac{n^2\pi^2}{p_e}-\frac{n\pi^2}{p_e}+\frac{\pi^2}{4p_e}$,

$m_3=\frac{1}{p_e}\left(\frac{2n-1}{2}\right)^2\pi^2$, $q_0=\frac{-2n^2\pi^2}{p_e}+\frac{2n\pi^2}{p_e}-\frac{\pi^2}{2p_e}+M+\frac{n^2\pi^2}{R_e}+n$,

$q_1=\frac{-n^2\pi^2}{p_e}+\frac{n\pi^2}{p_e}-\frac{\pi^2}{4p_e}+M+\frac{n^2\pi^2}{R_e}$, $q_2=M+\frac{n^2\pi^2}{R_e}+n$, $q_3=q_0\left(m_3-m_0\right)$,

$q_4=\frac{1}{q_0q_0}\left(\frac{q_0-q_1}{m_3-m_0}\right)$, $q_5=q_1\left(m_3-m_2\right)$, $q_6=m_1m_3$, $q_7=\frac{1}{q_2m_1}$,

$q_8=q_2\left(m_3+n\right)$, $q_9=m_3-m_1$

The computations were done using computer symbolic algebraic package MAPLE.

Results and Discussion

We solve the systems of coupled nonlinear partial differential equations describing Transient free convection magnetohydrodynamics flow between two heated vertical plates analytically using parameter-expanding method and eigenfunctions expansion technique. Analytical solutions of equations (6) - (8) are computed for the following parameter values: $M=1,\ P_r=0.71,\ R_e=1,\ a=0.001$. The following figures 2 – 5 display the temperature and velocity distributions against different dimensionless parameters.

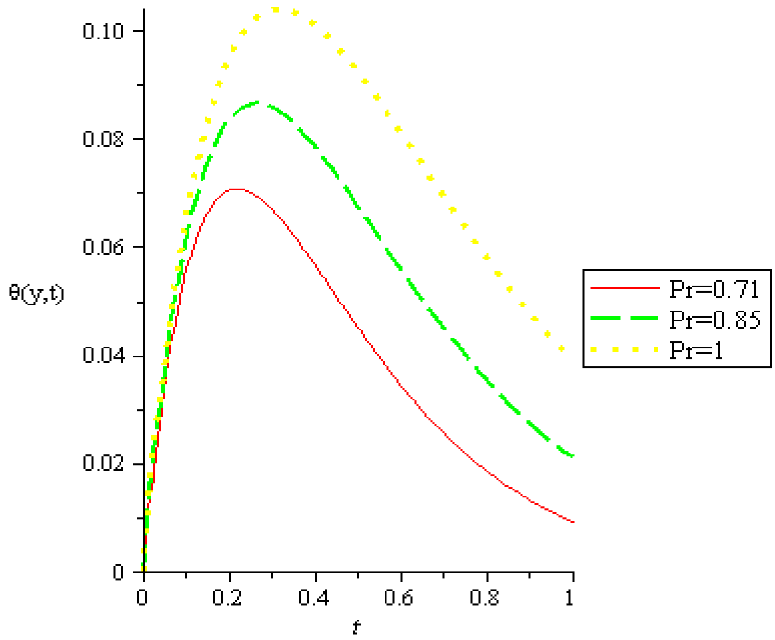

Figure 2: Variation of Temperature $\theta(y, t)$ with time t for different values of Pr

Figure 2 shows the graph of temperature against time for different values of Prandth number (Pr). It is observed that the temperature increases and later decreases with time while the maximum temperature increases as Prandth number increases.

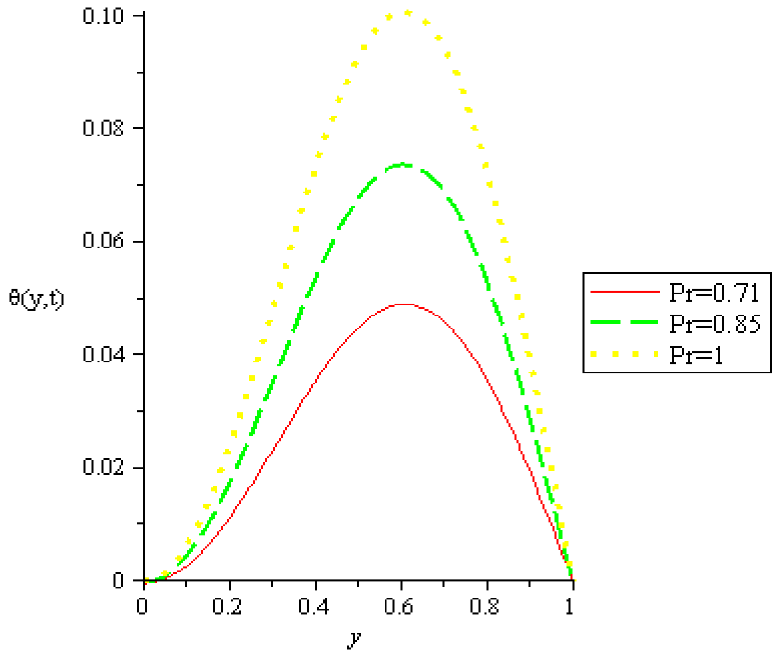

Figure 3: Variation of Temperature $\theta(y, t)$ with distance y for different values of Pr

Figure 3 depicts the graph of temperature against distance for different values of Prandth number (Pr). It is observed that the temperature increases and later decreases with along distance while the maximum temperature increases as Prandth number increases.

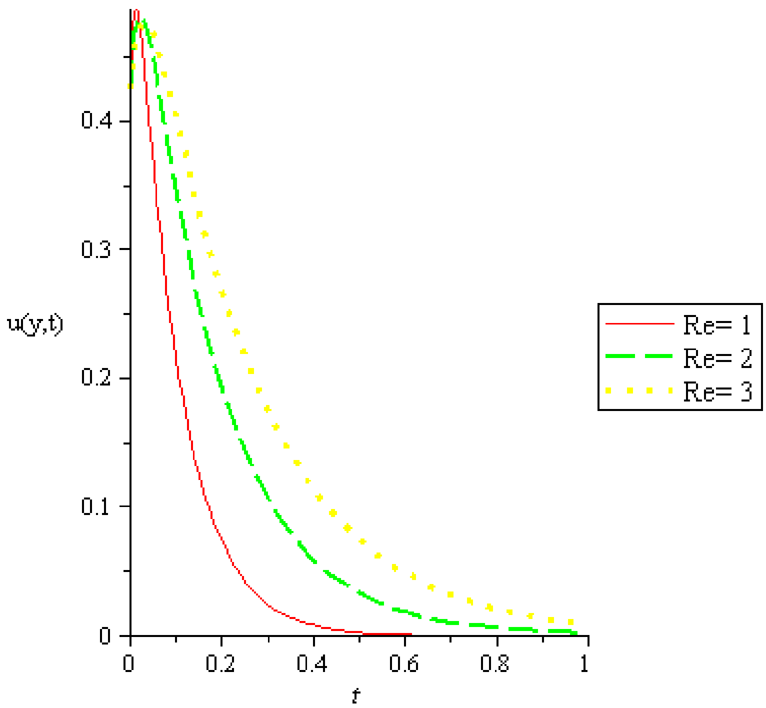

Figure 4: Variation of Velocity $u(y, t)$ with time t for different values of Re

Figure 4 displays the graph of velocity against time for different values of Reynolds number (Re). It is observed that the velocity decreases with time and increases as Reynolds number increases.

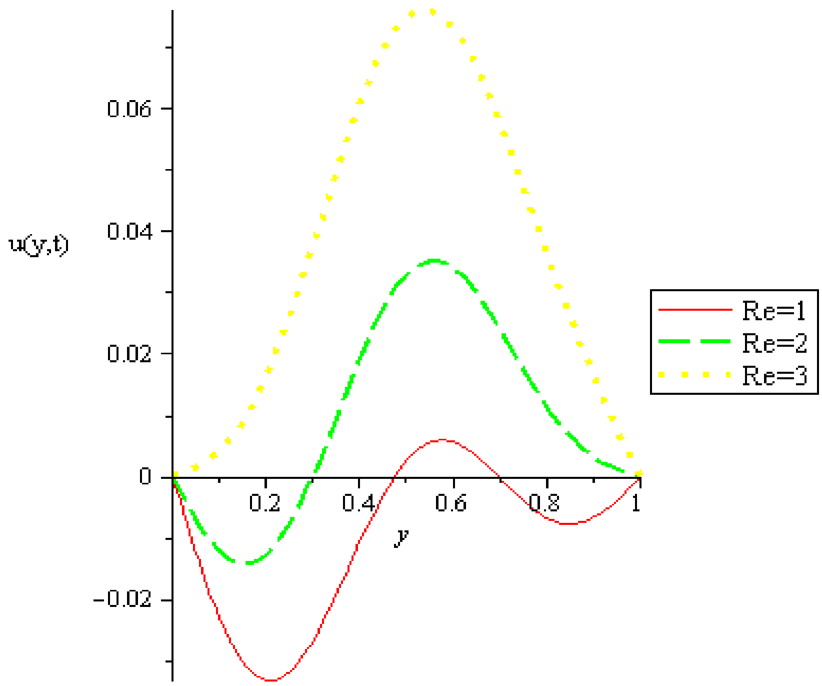

Figure 5: Variation of Velocity $u(y, t)$ with distance y for different values of Re

Figure 5 displays the graph of velocity against distance for different values of Reynolds number (Re). It is observed that the velocity oscillate along the distance and maximum velocity increases as Reynolds number increases.

Conclusion

We have formulated and solved analytically a mathematical model oftransient free convection magnetohydrodynamics flow between two heated vertical plates to determine the velocity and temperature distributions. We decoupled the equations using parameter expanding method and solved the resulting equations using eigenfunctions expansion technique. From the results obtained, we can conclude that:

(i) Prandth number enhances the temperature of the medium.
(ii) Reynolds number enhances the velocity of the flow.

References

Ashokkunar, K., Venna, P. H., & Pravin, V. K. (2013). Closed form solutions of heat and mass transfer in the flow of a MHD visco-elastic fluid over a porous stretching. *International journal of applied mechanics and engineering, 18*(4), 989-1002.

Bhaskar, K. (2014). Effects of mass transfer on unsteady free convection MHD flow between two heated vertical plates in the presence of transverse magnetic field. *International of Mathematical Engineering and Science,* 1(3), 1-13.

Chand, K., Kumar, R., & Sharma, S. (2012). Hydro magnetic oscillatory flow through a porous medium bounded by two vertical plate with heat source and soret effect. *Pelagia Research Library Advance in Applied Science Research,* 3(4), 2169-2178. www.pelaglaresearchlibrary.com Downloaded on 11/12/2014.

He, J. H. (2006). Some asymptotic methods for strongly nonlinear equations. *Int. J. Modern Phys.* B., 20(10), 1141 – 1199.

Ibrahim, S. M. (2014). Unsteady MHD convection heat and mass transfer past an infinite vertical plate embedded in a porous medium with radiation and chemical the reaction under influence of dufour and soret effects. *Chemical and process engineering research,* 19, 2014.

Lakshmi, K. B., Raju, G. S. S., Kishore, P. M., & Prasada-Rao, N. V. R. V. (2013). The study of heat generation and viscous dissipation on MHD and mass diffusion flow past a surface. *IOSR-Journal of Apply Physics*, *5*(4), 17-28.

Myint-U, T., & Debnath, L. (1987). *Partial differential equation for scientists and engineers.* New Jersey: PTR Prentice – Hall, Englewood Cliffs.

Olajuwon, B. I., & Dahimire, O. (2013). Unsteady free convection heat and mass transfer in an MHD micro polar fluid in the presence thermo diffusion and thermal radiation. *International Journal of and Pure Applied Mathematics, 84*(2), 15-37.

Oladeji, S. O. (2015). Magnetic field and heat dissipation effects on unsteady free convection Magnetohydrodynamics flow between two heated vertical plates. M.Tech. Thesis, FUT, Minna, Nigeria.