DEVELOPMENT OF A MATHEMATICAL MODEL FOR UPLAND RICE PRODUCTION (A CASE STUDY OF NATIONAL CEREAL RESEARCH INSTITUTE BADDEGI, NIGERIA)

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Abstract
In this paper, mathematical development of two quadratic models was used to obtain minimum, maximum and saddle points of yield response of upland rice production. The results showed that the models were adequate and significant at 5% by using canonical analysis through the methods of least square, R^2 (coefficient of determination) strength, R^2 -adjust, coefficient of variation (CV) and root mean square error (RMSE). The results also indicated that the quadratic effect of irrigation is important during the dry season than nitrogen for optimum yields. It was also established that there was an increase in yield when variance (ANOVA) was used for the data collected from National Cereal Research Institute Baddegi, Niger State. And computation of the data was adequate with R^2 above 60%, R^2 -adjusted above 55% and RMSE was very small.

Keywords: *Upland, Nerica Rice, Quadratic Model, Minimum, Maximum and Saddle Points, Yield Response and Canonical Analysis.*

Introduction
Finley (1972), said response surface methodology examines the relationship between several explained variables. And one or other necessary variables polynomial models are examined through the use of factorial experiment or a fractional factorial design. Response surface methodology is an important way of keeping records to help researchers improve products and services.

Cox (1958), said the application of Mathematics to the production of crops has gained dominance since rice production has become a global issue today through irrigational system and fertilizer application; the application of mathematical modeling becomes imperative. It is known that the mathematical examination of the impact on the objective of production at both rural and urban areas requires appropriate tool such as Mathematical modeling.

Friedman*, etal* (1948) said that mathematical modeling shows the understanding of Mathematics and help in upland rice production to give yield results. Mathematical models are useful examining tools for manipulating and testing theories, accessing quantitative variables, answering constructive questions, accessing sensitivities to changes in parameter values, and examining key parameters from data observed or collected.

Box, *et.al* (1990) said the closer Mathematical believes are to reality of behaviors, the more difficult the Mathematical analysis, hence the need to simplify our fillings without losing track of the situation or fillings at hand. Thus, the choice of using Mathematical modeling approach in this research work cannot be over stated.

Hakimi (2005), said the most appropriate model depends on the precision or generality required, the available data, and the time frame in which results are needed. It is therefore, difficult to express definitively which model is "right", though naturally we are interested in developing models that capture the essential features of a system. Ultimately, we are faced with the usefulness of any model.

This paper is aimed at developing a Mathematical model and analysis of a response surface outlook for upland rice production to meet up with the food consumption of people in Nigeria.

Materials and Methods

In this research work, we intend to know whether there was loss or increase in the yield result of rice production in the direction of application of irrigation system 'I'. Also, we intend to examine the coefficient of determination R^2, R^2 – adjusted, root mean square error (rmse) and coefficient of variation (CV) to check the model adequacy. So also the model equation formulated will be used to determine the point of maximum, minimum or a saddle point of the rice production.

Quadratic Fit Model

To develop the model equation for the farming system of rice production during the dry season of 2013 and 2014, all possible parameters were taken. We considered the irrigation system 'I', nitrogen fertilizer 'N', and seed varieties of rice 'V' to see the improvement of rice production during the dry season of the years mentioned above.

We also considered the interaction of irrigation system and nitrogen as 'IN', irrigation system and variety of rice as 'IV', nitrogen fertilizer and variety of rice as 'NV'.

The data we used for this research work were collected from NCRI Baddegi, Niger State. The data were based on the field trials during the third quarter of the years mentioned above. The treatments that were applied in the course of experimentation comprised of three (3) irrigation intervals (i.e 7, 14 and 21 days), four (4) fertilizer rates (i.e 30, 60 and 120kg Nha^{-1}) which were randomly allocated to the main plots (i.e the region of interest), while four (4) Nerica rice varieties (i.e 2, 3, 4 and 14) constituted the sub-plots.

The fitting modeled equation for the three factors is;

$$\gamma = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{11}x^2{}_1 + b_{22}x^2{}_2 + b_{33}x^2{}_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_1x_2 + \varepsilon \quad (1)$$

Where

γ = yield response

b_i = ϵstimated Parameters ($i = 0,1,2$ and 3)

x_1=I = Irrigation system

x_2=N = Nitrogen fertilizer applied

x_3=V = Variety of rice to be produced

ε = random error (i.e constant)

Complete Factorial versus Composite Design

For this research work, we used a complete factorial to produce a model of quadratic surface and it was used instead of a composite design.

A composite design has a 2^k+2k+1 treatment combination, and when we applied it to our research design, we had one factor at three levels i.e 2^1+2(1) + 1 = 5 and 2^2+2(2) + 1 = 9 for two factors at four levels. Therefore, the total treatment combinations will be 5x9 = 45.

On the other hand, for a complete factorial design, we had $3x4^2$factorial which is 3x4x4 = 48 treatment combinations. Combinational treatment for the composite design is reduced compared to the complete factorial design. Hence, we chose a complete factorial design for the following reasons:

(i) A full factorial approach will enable equal variance in the estimated effects.
(ii) It will also allow adequate degrees of freedom for error

Quadratic Surface for More than one Factor

Equation (1) gives a response function of a quadratic fit model for the three factors used in this research work.

$$\gamma - b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{11}x^2{}_1 + b_{22}x^2{}_2 + b_{33}x_3{}^2 + b_{12}x_1x_2 + b_{13}x_1x_3$$
$$+ b_{23}x_2x_3 + \varepsilon$$

Step 1:

The surface is obtained by taking the partial derivatives with respect to x_1,x_2, $x_3$and setting them equal to zero, we had

$$\frac{\partial\gamma}{\partial x_1} = b_1 + 2b_{11}x_1 + b_{12}x_2 + b_{13}x_3 = 0 \quad (2)$$
$$\frac{\partial\gamma}{\partial x_2} = b_2 + b_{12}x_1 + 2b_{22}x_2 + b_{23}x_3 = 0 \quad (3)$$
$$\frac{\partial\gamma}{\partial x_3} = b_3 + b_{13}x_1 + b_{23}x_2 + 2b_{33}x_3 = 0 \quad (4)$$

The solutions of these equations give the factorial combinations at which y is a local maximum, minimum or a local stationary value.

Canonical Analysis

We applied canonical analysis to determine whether the stationary point is a maximum, minimum or saddle point. Contour plots may also be used.

Step 2:

Solving equation (2), (3) and (4) to obtain the points of optimum for x_1, x_2 *and* x_3

$$\left.\begin{array}{l} b_1 + 2b_{11}x_1 + b_{12}x_2 + b_{13}x_3 = 0 \\ b_2 + b_{12}x_1 + 2b_{22}x_2 + b_{23}x_3 = 0 \\ b_3 + b_{13}x_1 + b_{23}x_2 + 2b_{33}x_3 = 0 \\ \\ 2b_{11}x_1 + b_{12}x_2 + b_{13}x_3 = -b_1 \\ b_{12}x_1 + 2b_{22}x_2 + b_{23}x_3 = -b_2 \\ b_{13}x_1 + b_{23}x_2 + 2b_{33}x_3 = -b_3 \end{array}\right\} \quad (5)$$

We write the above system of equation in Matrix form

$$Ax = b \quad (6)$$

$$\begin{pmatrix} 2b_{11} & b_{13} & x_{13} \\ b_{12} & 2b_{22} & b_{23} \\ b_{13} & b_{23} & 2b_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -b_1 \\ -b_2 \\ -b_3 \end{pmatrix} \quad (7)$$

By using Cramer's Rule we obtained the value of x_1, x_2 and x_3 as follows:
Let

$$\Delta_1 = \begin{pmatrix} 2b_{11} & b_{12} & b_{13} \\ b_{12} & 2b_{22} & b_{23} \\ b_{13} & b_{23} & 2b_{33} \end{pmatrix} \tag{8}$$

$$\det\Delta_1 = \begin{pmatrix} 2b_{11} & b_{12} & b_{13} \\ b_{12} & 2b_{22} & b_{23} \\ b_{13} & b_{23} & 2b_{33} \end{pmatrix}$$

We solved and arrived

$$x_1 = \frac{(\Delta_2)}{(\Delta_1)} = \frac{\Delta_2}{\det \Delta_1} \tag{9}$$

$$x_1 = \frac{-4\, b_1\, b_{22} + b_1\, b_{23}\, b_{23} + 2\, b_2\, b_{12}\, b_{33} - b_3\, b_{12}\, b_{23} - b_2\, b_{13}\, b_{23} + b_3\, b_{13}\, b_{22}}{8b_{11}\, b_{12}\, b_{13} - 2\, b_{11}\, b_{13}\, b_{23} - 2\, b_{12}\, b_{12}\, b_{33} - b_{12}\, b_{12} + b_{12}\, b_{13}\, b_{23} + 2\, b_{13}\, b_{13}\, b_{22}} \tag{10}$$

$$x_2 = \frac{(\Delta_3)}{(\Delta_1)} = \frac{\Delta_3}{\det \Delta_1} \tag{11}$$

$$x_2 = \frac{-4\, b_1\, b_{11}\, b_{33} + 2b_3\, b_{11}\, b_{23} + 2\, b_1\, b_{12}\, b_{33} - b_1\, b_{13}\, b_{23} - b_3\, b_{13}\, b_{12} + b_2\, b_{13}\, b_{13}}{8b_{11}\, b_{12}\, b_{13} - 2\, b_{11}\, b_{23}\, b_{23} - 2\, b_{12}\, b_{12}\, b_{33} + b_{12}\, b_{12} + b_{12}\, b_{13}\, b_{23} + 2\, b_{13}\, b_{13}\, b_{22}} \tag{12}$$

$$x_3 = \frac{(\Delta_4)}{(\Delta_1)} = \frac{\Delta_4}{\det \Delta_1} \tag{13}$$

$$x_3 = \frac{-4\, b_3\, b_{11}\, b_{22} + 2b_2\, b_1\, b_{23} + b_3\, b_{12}\, b_{12} - b_2\, b_{12}\, b_{13} - b_1\, b_{12}\, b_{23} + 2b_1\, b_{13}\, b_{22}}{8b_{11}\, b_{12}\, b_{13} - 2\, b_{11}\, b_{23}\, b_{23} - 2\, b_{12}\, b_{12}\, b_{33} + b_{12}\, b_{12} + b_{12}\, b_{13}\, b_{23} + 2\, b_{13}\, b_{13}\, b_{22}} \tag{14}$$

Step 3:
To obtain the optimum response y_m (m is the optimum) we substitute the values of x_1, x_2 and x_3 in to equation (3) below;

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{11}x^2{}_1 + b_{22}x^2{}_2 + b_{33}x^2{}_3 + b_{12}x_1b_{13}x_1x_3 \\ +b_{23}x_{23}x_2x_3 + e \tag{15}$$

Step 4:
A determinant matrix is formed to determine the coefficients in the canonical form i.e

$$\begin{pmatrix} 2b_{11} & b_{12} & b_{13} & x_1 \\ b_{12} & 2b_{22} & b_{23} & \\ b_{13} & b_{23} & 2b_{33} & \end{pmatrix} \begin{pmatrix} -b_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -b_2 \\ -b_3 \end{pmatrix}$$

i.e

$$\begin{pmatrix} 2b_{11} & b_{12} & b_{13} \\ b_{12} & 2b_{22} & b_{23} \\ b_{13} & b_{23} & 2b_{33} \end{pmatrix} \tag{16}$$

We divided R_1, R_2 and R_3 by 2 and we had

$$\begin{pmatrix} b_{11} & \frac{b_{12}}{2} & \frac{b_{13}}{2} \\ \frac{b_{12}}{2} & b_{22} & \frac{b_{23}}{2} \\ \frac{b_{13}}{2} & \frac{b_{23}}{2} & b_{33} \end{pmatrix}$$

We had our characteristics equation as:

$$\begin{pmatrix} (b_{11}-\lambda) & \frac{b_{12}}{2} & \frac{b_{13}}{2} \\ \frac{b_{12}}{2} & (b_{22}-\lambda) & \frac{b_{12}}{2} \\ \frac{b_{13}}{2}\frac{b_{23}}{2} & (b_{33}-\lambda) & \end{pmatrix} = 0 \qquad (17)$$

Therefore, we solved and arrived at characteristics equation as:

$$-8\lambda^3 + (8b_{11} + 8b_{22} + 8b_{33})\lambda^2 - (2b_{12}b_{12} + 2b_{13}b_{13} + 2b_{23}b_{33} - 8b_{11}b_{22} - 8b_{11}b_{33} + 8b_{22}b_{23})\lambda - 6b_{11}b_{22}b_{33} + 2b_{11}b_{23}b_{23} - 2b_{12}b_{13}b_{23} + 2b_{13}b_{13}b_{22} - 0 \qquad (18)$$

Step 5

We substituted the values of x_1, x_2 *and* x_3 by using computer Algebraic symbolic package "Marple software", and subtracted the result from the original equation (i.e equation 3) to have.

$$y - y_m = b_{11}x^2{}_1 + b_{11}b^2{}_1 + b_{22}x^2{}_2 + b_{33}x^2{}_3 \qquad (19)$$

Results

The result obtained in step 5; y-y_m = $b_{11}x^2{}_1 + b_{22}x^2{}_2 + b_{33}x^2{}_3$ can be interpreted as a change in yield from the point of optimum, m to some points ($(x_1, x_2$ *and* $x_3)$.

The following reasons are observed:

If the coefficients $b_{11}.b_{12}$ are all negative, then there is a loss in yield whichever way one goes from m.

i. If b_{11}, b_{12} are negative and b_{33} is zero, there is still a loss in yield.
ii. If b_{11} is negative and b_{22} and b_{33} are both zero, there is no unique centre and we have a stationary ridge.
iii. If b_{11} is negative, b_{22} *and* b_{33} are both positive and measure an increase in yield along the x_2 *or* x_3 axis at infinity, then we have a rising ridge.

Fitting Model for 2013 Rice Yield

The model that was fitted for rice yield in 2013 is:

$$\gamma = 26.5 + 0.1x_1 - 0.02x_1{}^2 + 0.54x_2 - 0.01x_2{}^2 + 0.000013x_2{}^3 - 0.098x_1x_2 + 0.0014x_1x_2{}^2 - 0.000004x_1x_2{}^3 + 0.004x_1{}^2x_2 - 0.0001x_1{}^2x_2{}^2 + 0.00000021x_1{}^2x_2{}^3 \qquad (20)$$

where,

γ = response estimate for yield

x_1 = irrigation linear effect

$x_1{}^2$= irrigation quadratic effect

x_2 = nitrogen linear effect

$x_2{}^2$= nitrogen quadratic effect

$x_2{}^3$= nitrogen cubic effect

Equation (20) is the response surface polynomial for 2013 rice yield. The ANOVA table below helps us to select the parameters that are significant and needed for the response surface model.

Table 4.1: ANOVA table for 2013 rice yield

SOURCE	Degree of Freedom (df)	Sum of Square (SS)	Mean Squares (MS)	F	P-VALUE
REP	1	17.17	17.17	2.10	0.1508
x_1	2	742.95	317.47	45.52	
x_1	1	741.84	741.84	90.91	0.0001
x_1^2	1	1.11	1.11	0.136	0.7134
x_2	3	198.15	66.05	8.09	
x_2	1	193.40	193.40	23.70	0.0001
x_2^2	1	4.75	4.75	0.58	0.4479
x_2^3	1	0.0002	0.0002	0.000025	0.9957
x_1x_2	6	156.62	26.10	3.198	
x_1x_2	1	38.15	38.15	4.67	0.0335
$x_1x_2^2$	1	24.15	24.15	2.96	0.0892
$x_1x_2^3$	1	2.51	2.51	0.31	0.5805
$x_1^2x_2$	1	85.50	85.50	10.47	0.0017
$x_1^2x_2^2$	1	5.55	5.55	0.68	0.4119
$x_1^2x_2^3$	1	0.76	0.76	0.09	0.7613
ERROR	83	677.66	8.16		
TOTAL	95	1792.54			

Source: (Author, 2016)

From the table above it can be seen that at 5% level of significance the parameters x_1x_2, x_1x_2, and $x_1^2x_2$ are significant since their p-values are less than a = 5% significance level and our final response surface function is;

$$\gamma = 26.5 + 0.25x_1 + 0.54x_2 - 0.098x_1x_2 + 0.004x_1^2x_2 \qquad (21)$$

From the ANOVA table above we observed that x_1^2, x_2^2 and $x_1^2x_2^2$ are not significant at 5% level of significant, therefore, we concluded that effects on rice yield are not significant.

Discussion

Here we will determine whether the stationary point is a point of maximum, minimum or a saddle point.

The fitted model to be used is equation

$$\gamma = 26.5 + 0.25x_1 + 0.54x_2 - 0.098x_1x_2 + 0.004x_1^2x_2 \qquad (22)$$

Differentiate equation (4.2) partially w.r.t. x_1 and x_2 and equate to zero to find the optimum point we have;

$$\frac{\partial \gamma}{\partial x_1} = 0.25 - 0.098x_2 + 0.008x_1x_2$$

$$0 - 0.25 - 0.098x_2 + 0.008x_1x_2$$

$$0.098x_2 - 0.008x_1x_2 = 0.25 \qquad (23)$$

$$0 = 0.54 \quad 0.098x_1 \mid 0.004x_1^2$$

$$0 = 0.54 - 0.098x_1 + 0.004x_1^2$$

$$0.098x_1 - 0.004x_1^2 = 0.54 \qquad (24)$$

Solving simultaneously equation (1.23) and (1.24), we have;

x_1 = 16.13 or 8.37

Substituting x_1 = 16.13 and 8.37 into equation (1.24) to obtain values for x_2, we have;

x_1= - 8.051 and 8.054 when x_1 = 16.13 and 8.369 respectively; these are the optimum points, that is, x_1 = 16, x_2 = 0 or x_1 = 8, x_2 = 8.

We then substitute $x_1$and x_2 using x_1 = 16, x_2 = 0 and x_1 = 8, x_2 = 8.05 in equation (1.20) to obtain the optimum response γ_m as;

γ_m= 26.5 + 0.25(16) + 0.54(0) – 0.098(16)(0) (0.004*0)(16)^2

γ_m= 30.5 is the optimum response for x_1 = 16 x_2 = 0

γ_m= 26.5 is the optimum response for x_1 = 8 and x_2 = 8

We therefore constructed a determinant matrix as follow to determine the coefficient in the canonical form.

$$\begin{vmatrix} b_{11} - \lambda & (b_{12})/2 \\ \frac{b_{12}}{2} & b_{22} - \lambda \end{vmatrix}$$

Determine the Eigen-values or characteristic roofs and equate the determinant to zero, we have;

$$\begin{vmatrix} 0 \quad \lambda & 0.049 \\ -0.049 & 0 - \lambda \end{vmatrix} = 0$$

Solving the determinant matrix we have;

$-\lambda(-\lambda) - (-0.049 * -0.049) = 0$

$\lambda^2 = 0.002401$

$\lambda = \pm 0.049$

taking the values of λ we have two set of canonical equations as

$$\gamma - 30.5 = 0.049x_1^2 - 0.049x_2^2 \tag{25}$$

$$\gamma - 33.3 = 0.049x_1^2 - 0.049x_2^2 \tag{26}$$

Equation (1.25) is obtained when x_1= 16 and x_2= 0 while, equation (1.26) is obtained when x_1 = 8 and x_2= 8. We observed that the contours of both equation (1.25) and equation (1.26) are saddle surfaces because their coefficients b_{11} andb_{22} are positive and negative respectively, which means that there will be a rapid increase in yield if $x_1$is increased. This shows that there will be an increase in yield in the direction of x_1 axis from the optimum response *M*.

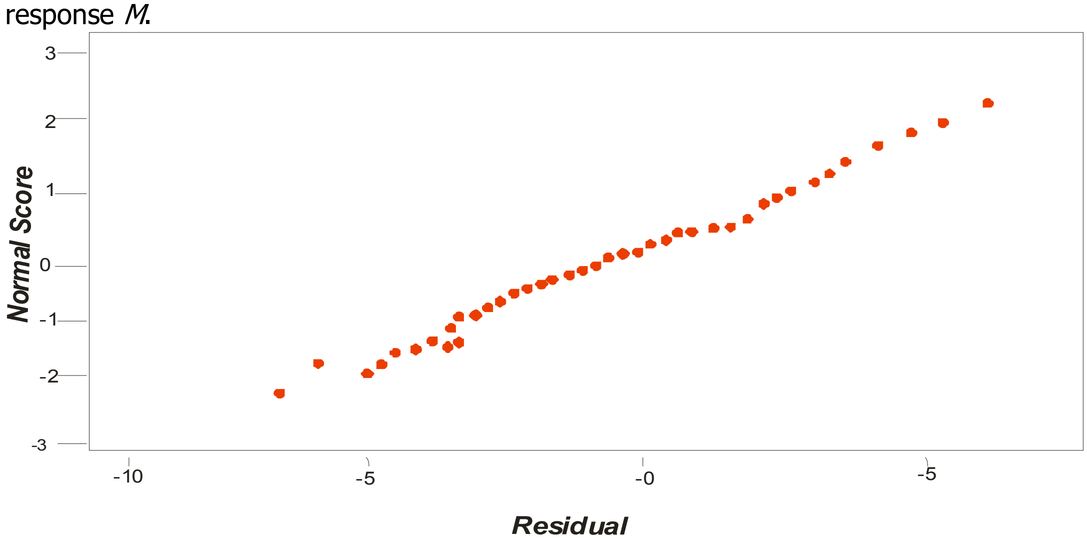

Figure1: Normal probability plot of the residuals for 2013 rice yield

Figure 1 above is the normal probability plot for 2013 rice yield data to check for normality of the data and it appears that the shape confirms the normality of the data.

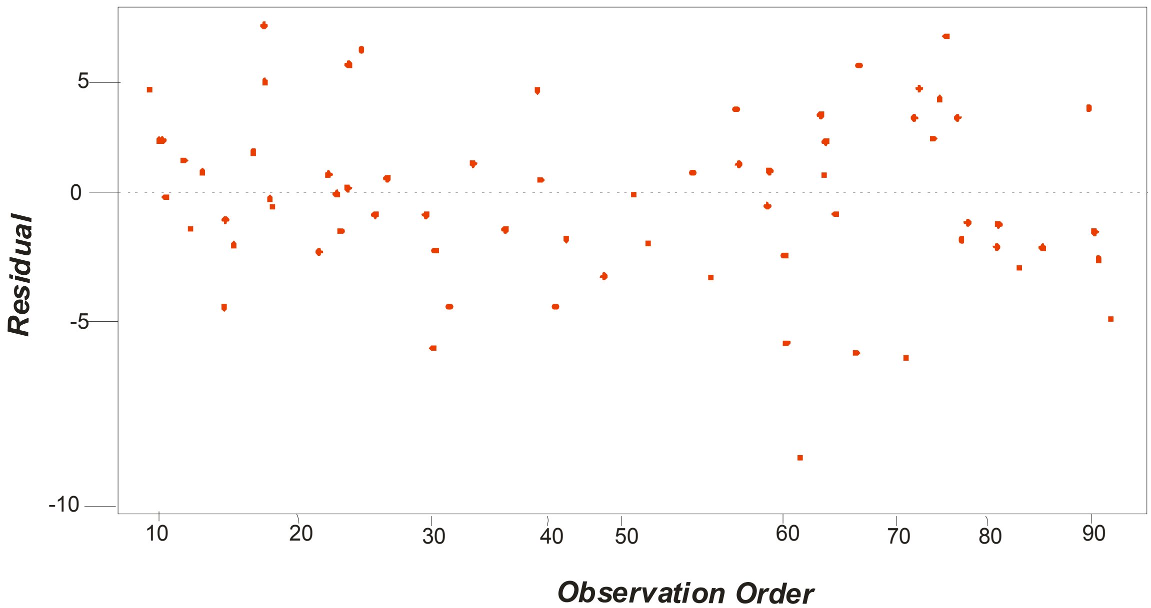

Fig 2: Residuals versus the order of the data for 2013 rice yield

Figure 2 above is a graph showing the residuals versus the order of the data for 2013 rice data it is a confirmatory graph of figure 4.2 (residual versus fitted values).

Conclusion

Basically, a complete factorial experiment where each factor of all the levels are absolutely combined and the model was adequate. We found that the linear and quadratic response surfaces were significant. The model showed that in the optimum response region, there was an increase in yield when irrigation and nitrogen are combined at quadratic and linear effect levels respectively.

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