MAXIMIZATION OF FLOW RATE IN A WATER DISTRIBUTION NETWORK USING LINEAR PROGRAMMING MODEL

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Abstract
The application of linear programming model to maximize water supply in a water distribution network has been presented in this paper. Thus, this paper proposes a new flow rate to maximize the flow of water in a community to meet the demand of the residents. The network of the pipes connecting the Water Treatment Plant and the Reservoir in the community were both considered. The problem was transformed into a linear programming problem, then solved using a computer software. A new flow rate of 0.03681 m^3/s with a volume of 3180.38m^3 per day was obtained against the old flow rate of 0.02778m^3/s with a volume of 2400.192m^3 per day. The result of the study is an important information to the management of Water Works of the community in boosting water supply to the residents.

Keywords: Water, maximize, linear programming, flow rate

Introduction

The Maximum-flow problem for water distribution system network was considered in the study. The Maximum-flow problem is a problem that maximizes the total amount of flow of water from the source to the sink. It entails finding a maximum possible flow through a single-source, single-sink flow network. The increase expense of investment for different Water Distribution System elements makes long-term network planning an important problem(Kekatos & VassilisSingh, 2019). Lack of sufficient water in the community is gradually becoming an all-time high. It has become a source of concern to water distribution managers, water resource management scientists and the public.
The most important thing for the continual existence of life on earth after air is water, it is on this note that this research is been focused on water supply. The importance of water cannot be over-emphasized, especially in human life. It has been observed that water has not been properly distributed in the community under study which has consequently affect the demand of the residents, hence the aim of this research is to propose maximum flow rate for the water distribution network of Bosso Zone E1 community, Niger state, Nigeria within the budgeted cost with a view to improving water supply to community. Many researchers around the world have tried to assist water management scientists with information to boost water supply to the residents among them are: Mohammad and Samani (2015), in this study, optimum design of municipal water distribution networks was presented using integer linear programming. The objective function was to obtain the optimum design, which has the least total cost. Presented in (D'Ambrosio *et al.*, 2015) is a water network optimization problem that retained the projection or uncommon structure for which all sub-problems had fixed integer variables. The overall mathematical model was a Mixed Integer Nonlinear Program having a common structure regarding how water dynamics in pipes is described. Dimri and Ram(2018) proposed that splitting of the flow in smaller paths is a logical strategy that could be used to reduce the delay, to maximize the amount of flow and for optimum network resource utilization. Vieira *et al.*, (2018) proposed a new linear relaxation for a non-linear integer programming formulation for Water

Distribution Systems in order to optimize its operation costs. Comprehensive study of water distribution network has also been reported in the following researches: (Ameyaw *et al*., 2013), (Bello *et al.,* 2015), (Bonvin *et al.,* 2017), (Candelieri *et al.,* 2018), (Chen *et al.,* 2019),(Kurian *et al.,* 2018), (Zhang & Zhuan, 2019).

Materials and Methods

The data used in this research was collected from Bosso Water Works, Niger State, Nigeria. The record contains the detailed network flow of the water distribution system of the community as show in Figure1.

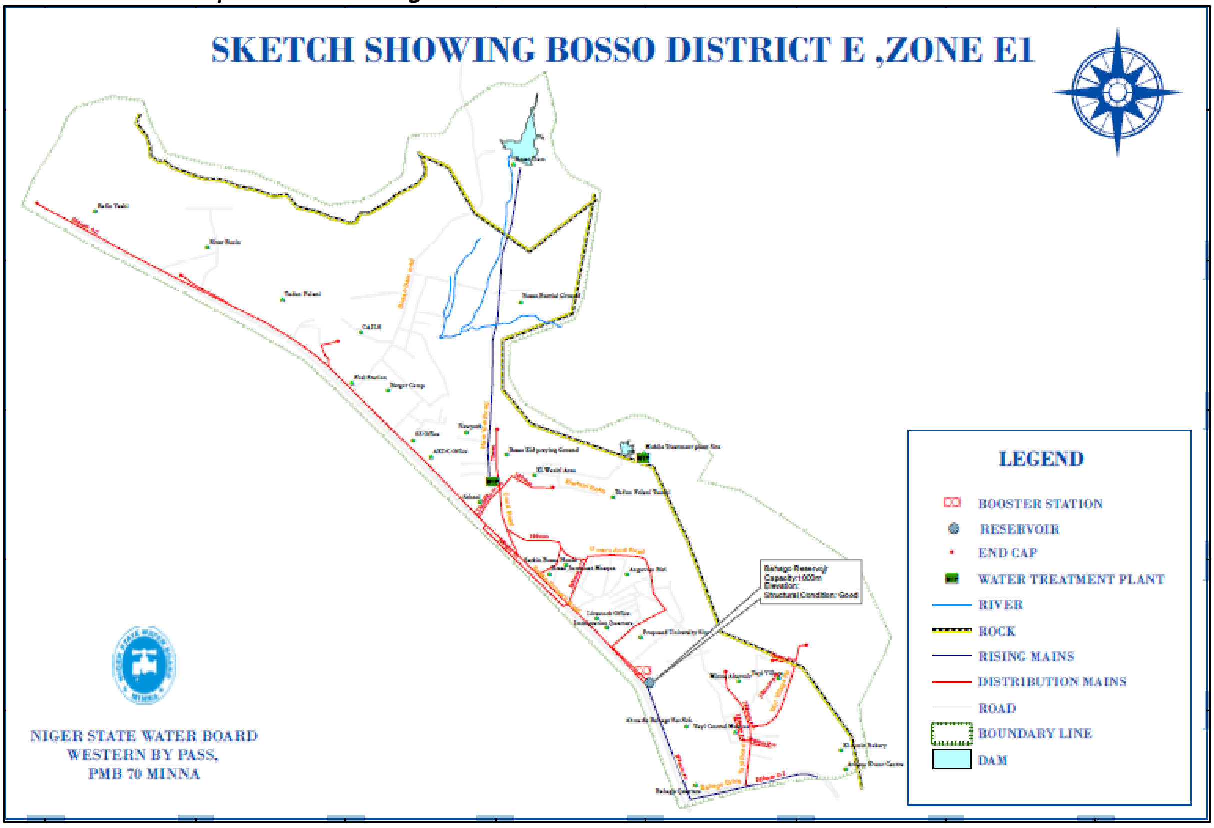

Figure1: Sketch of Bosso District, Zone E1
(Niger State Water Board)

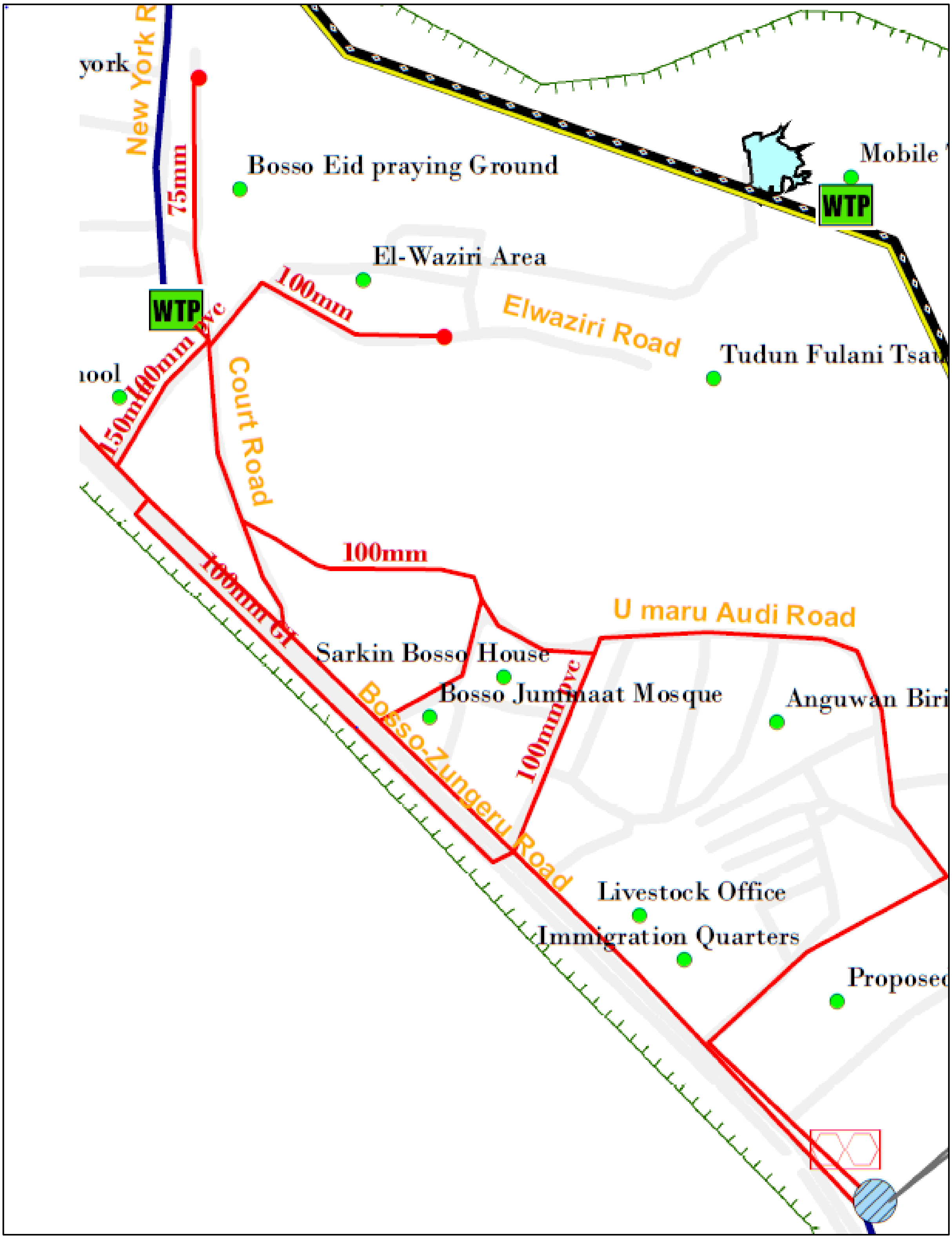

Figure 2: Water Distribution Network of Bosso, Zone E1 (Niger State Water Board)

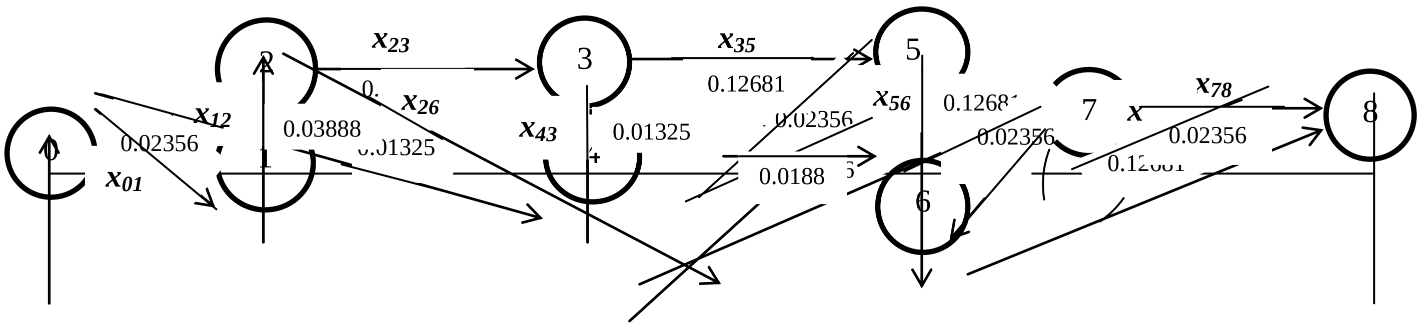

Figure 3: Bosso Zone E1, Water Distribution Network with Associated Flow Rate Based on the pipes Capacities (Extracted from Figure 2)

Table1: Capacity of pipes used and their flow rate

SIZE OF PIPES	PIPE CAPACITIES OR FLOW RATE (m^3/s)
75mm	0.01325306387
100mm pvc	0.02356123805
100mm G I	0.01883824788
150mm	0.03887643155
300mm A C	0.1268107409

Formulation of the Linear Programming Model

Figure 3 is the basic water distribution network of Bosso zone E1 extracted from Figure2 with associated flow rate calculated based on pipe capacities used in the network.

Figure 3, is a directed graph G (V,E) with a set of vertices V and a set of edges E. There are two distinguished nodes (vertices):the source, s and the sink, t. Each edge has an associated capacity c_{ij} for every edge $(ij) \in E$ In our formulation, nodes do not produce or consume flow, but rather, we introduce an auxiliary edge (s, t) with no capacity limit and we aim to maximize flow along this edge. By doing so, we indirectly maximize flow from t to s through the edges in E. This flow maximization problem is then transformed into a linear programming problem with the objective of maximizing total flow between S and T with the restriction of the edges capacities, i.e. the flow value in an edge cannot exceed the capacity of the edge and the total flow cost cannot be higher than the given budget. The flow variables are obtained from the directed graph as: x_{01}, x_{04}, x_{12}, x_{23}, x_{26}, x_{43}, x_{45}, x_{47}, x_{56}, x_{76}, x_{78}, x_{68}, x_{35}, x_{80} these represent the flow from node 0 to node 4 as x_{04}, the flow from node 1 to node 2 as x_{12}, the flow from node 2 to node 3 as x_{23}, the flow from node 2 to node 6 as x_{26}, the flow from node 4 to node 3 as x_{43}, the flow from node 4 to node 5 as x_{45}, the flow from node 4 to node 7 as x_{47}, the flow from node 5 to node 6 as x_{56}, the flow from node 7 to node 6 as x_{76}, the flow from node 7 to node 8 as x_{78}, the flow from node 6 to node 8 as x_{68}, the flow from node 3 to node 5 as x_{35} and flow from node 8 to node 0 as x_{80} respectively. Here, the variables are the flows on the edges. We have two types of constraints in this network which are Capacity constraints and Flow conservation constraints.

(i) For the Capacity constraints:

The flow over any link cannot exceed the capacity of that link, ∀ link e: $f_e \leq c_e$ which are:

With constraints $x_{01} \leq 0.02356123805$, $x_{04} \leq 0.01325306387$, $x_{12} \leq 0.03887643155$, $x_{23} \leq 0.1268107409$, $x_{26} \leq 0.01883824788$,$x_{35} \leq 0.1268107409$, $x_{43} \leq 0.01325306387$, $x_{45} \leq 0.02356123805$, $x_{47} \leq 0.02356123805$, $x_{56} \leq 0.1268107409$, $x_{68} \leq 0.1268107409$, $x_{76} \leq 0.02356123805$, $x_{78} \leq 0.02356123805$.

(ii) For the Flow conservation constraints:

The flow conservation constraint is valid for nodes other than the source and the sink. Total flow flowing into a node = Total flow flowing out of a node
i.e. ∀ node n (n ≠ S and n ≠ T): ∑ (flow into n) = ∑ (flow out of n).

We then introduce an artificial flow from sink T back to source S which is from the Water Treatment Plant (source) to the Reservoir (sink). We then represent the nodes as variables. From Figure3, let Water Treatment Plant be node 0, School be node 1, Along Bosso - Zungeru Road be node 2, Junction be node 3, Bosso Court be node 4, Bosso Junimaat Mosque Junction be node 5, Immigration Quarters Junction be node 6, Umaru Audi Road Junction be node 7, Reservoir be node thus, we denote the artificial flow from node i to node j as x_{ij}

Thus, we have the flow variables: x_{01}, x_{04}, x_{12}, x_{23}, x_{26}, x_{43}, x_{45}, x_{47}, x_{56}, x_{76}, x_{78}, x_{68}, x_{35}, x_{80}. From the flow conservation constraints, we have that the flow into source = flow out of source as represented by equation

$$X_{80} = x_{01} + x_{04} \quad (1)$$

The flow conservation constraints are:

$$\left.\begin{aligned} x_{01} &= x_{12} \\ x_{12} &= x_{23} + x_{26} \\ x_{23} &= x_{35} \\ x_{04} &= x_{43} + x_{45} + x_{47} \\ x_{35} + x_{45} &= x_{56} \\ x_{56} + x_{76} &= x_{68} \\ x_{47} &= x_{76} + x_{78} \end{aligned}\right\} \quad (2)$$

where $x_{01} \leq 0.02356123805$, $x_{04} \leq 0.01325306387$, $x_{12} \leq 0.03887643155$, $x_{23} \leq 0.1268107409$, $x_{26} \leq 0.01883824788$,$x_{35} \leq 0.1268107409$, $x_{43} \leq 0.01325306387$, $x_{45} \leq 0.02356123805$, $x_{47} \leq 0.02356123805$, $x_{56} \leq 0.1268107409$, $x_{68} \leq 0.1268107409$, $x_{76} \leq 0.02356123805$, $x_{78} \leq 0.02356123805$.

Thus we have the linear programming model as:

Maximize $Z = x_{01} + x_{04}$ (3)

subject to flow constraints: $x_{01} = x_{12}$

$$\left.\begin{aligned} x_{12} &= x_{23} + x_{26} \\ x_{23} &= x_{35} \\ x_{04} &= x_{43} + x_{45} + x_{47} \\ x_{35} + x_{45} &= x_{56} \\ x_{56} + x_{76} &= x_{68} \\ x_{47} &= x_{76} + x_{78} \end{aligned}\right\} \quad (4)$$

with x_{01} ≤ 0.02356123805, x_{04} ≤ 0.01325306387, x_{12} ≤ 0.03887643155, x_{23} ≤ 0.1268107409, x_{26} ≤ 0.01883824788, x_{35} ≤ 0.1268107409, x_{43} ≤ 0.01325306387, x_{45} ≤ 0.02356123805, x_{47} ≤ 0.02356123805, x_{56} ≤ 0.1268107409, x_{68} ≤ 0.1268107409, x_{76} ≤0.02356123805, x_{78} ≤ 0.02356123805.

Results and Discussion

We proceed to solve the linear programming problem by expressing it in its standard form as:

Maximize Z $-x_{01} - x_{04} - 0x_{12} - 0x_{23} - 0x_{35} - 0x_{43} - 0x_{45} - 0x_{47} - 0x_{56} - x_{68} - 0\,x_{76} - 0x_{78} = 0$ (5)

Subject to
$$\left.\begin{aligned} x_{01} - x_{12} &= 0 \\ x_{12} - x_{23} - x_{26} &= 0 \\ x_{23} - x_{35} &= 0 \\ x_{04} - x_{43} - x_{45} - x_{47} &= 0 \\ x_{35} + x_{45} - x_{56} &= 0 \\ x_{56} + x_{76} - x_{68} &= 0 \\ x_{47} - x_{76} - x_{78} &= 0 \end{aligned}\right\} \quad (6)$$

where x_{01} ≤ 0.02356123805, x_{04} ≤ 0.01325306387, x_{12} ≤ 0.03887643155, x_{23} ≤ 0.1268107409, x_{26} ≤ 0.01883824788, x_{35} ≤ 0.1268107409, x_{43} ≤ 0.01325306387, x_{45} ≤ 0.02356123805, x_{47} ≤ 0.02356123805, x_{56} ≤ 0.1268107409, x_{68} ≤ 0.1268107409, x_{76} ≤0.02356123805, x_{78} ≤ 0.02356123805.

Solving the above equation using LiPS computer software, thus we obtain the solution below

(x_{01}, x_{04}, x_{12}, x_{23}, x_{26}, x_{35}, x_{43}, x_{45}, x_{47}, x_{56}, x_{68}, x_{76}, x_{78}) = (0.02356, 0.01325, 0.02356, 0.00472, 0.01884, 0.00472, 0.01325, 0, 0, 0.00472, 0, 0, 0.00472).

The optimal value for the objective function is Z = 0.03681m^3/s.

Thus, the maximum value gotten is the maximum flow that the system can accommodate within the budgeted cost. That is the maximum flow that can be gotten from the system as compared to the former rate of 0.02778m^3/s.

Conclusion

At the end of our computation, we obtained our optimal solution as 0.03861m^3/s. This shows an increase in supply though the total demand is not fully met. The facility has not been able to meet the demands of the people in the community. The daily demand is estimated to be about 10000m^3 daily. With this increase flow rate, which is 3180.38cubic metres per day against the old flow rate of 0.02778m^3/s with a volume of 2400.192 cubic metre per day, Bosso Water Works should be able to provide more water to its community.

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