MODELING FIRE SPREAD BEHAVIOUR IN COUPLED ATMOSPHERIC-FOREST FIRE

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Abstract
In this study, a mathematical model governing Coupled Atmospheric-Forest Fire, which is based on balance equations for energy and fuel is presented. The equations governing this phenomenon were solved analytically via direct integration and eigenfunction expansion technique. The obtained results depict the effects of parameters involved on the system. It is observed that the activation energy number reduced the transient medium temperature while it enhanced the oxygen concentration.

Keywords: Bush fire, combustible, fire behavior, fire spread, forest fire, modeling and surface fire

Introduction
Forest fire spread is one of the most challenging problems faced by reserved or unreserved vegetation in many developed and developing nations, because it can lead to serious environmental hazards in claiming lives, properties, animals and some other valuable resources. Forest fire also known as rural or bush fire refers to an uncontrolled or unwanted fire in an area of combustible vegetation occurring most likely in rural areas (Scott & Glasspool, 2006). The forest fires are a common occurrence in most parts of the world and they cause a lot of havoc to biodiversity as well as to the local ecology. Forest fires impact millions of lives and difficult to combat in nature, yet fire spread cannot be done away with, but can only be controlled or managed. Forest fire ignition could be as a result of; human action (intentional) in clearing of land, extreme/intensive drought, in rare cases thunderstorm (lightning) and hunter's burning bush in search of wild animals. For both human, extreme/intensive drought and lightning-caused fires, there is a geographical gradient of fire ignition, mainly due to variations in climate and fuel composition but also to population density for instance. The timing of fires depends on their causes. In populated areas, the timing of human-caused fires is closely linked to human activities and peaks in the afternoon whereas, in remote areas, the timing of lightning-caused fires is more linked to weather conditions and the season, with most such fires occurring in summer. Better tools for modelling forest fire behavior are important for managing fire suppression, planning controlled burns to reduce the fuels, as well as to help assess fire danger (Anne *et al.,* 2012).

A great deal of work has been carried out on the how bush fire spread (Perminov, 2005). Forest fire models have been developed since 1940 to the present, but a lot of chemical and thermodynamic questions related to fire behavior are still to be resolved. Forest fires are divided into underground (peatbog) fires, surface fires (ground fires), active crown fires, running crown fires (also called independent crown fires), crown fires and mass fires (Grishin, 2002). The forest fires are usually a common occurrence in most parts of the world and capable of making the habitat unfit for biotic and abiotic components of the ecosystem. Fire models range from tools based on fire spread rate formulas, such as BehavePlus (Rothermel, 1972; Andrews, 2007). There have been several researches conducted on vegetation fire to understand and clarify the consequences and then plans made to mitigate these vegetation fires. Perminov (2018) in his demonstration describes forest fires as a complicated phenomenon and at present, fire services can forecast the danger rating of or the specific weather elements relating to forest fire. There is a need to understand and predict forest fire initiation, behavior and spread.

It looks more interesting to use methods of mathematical modeling that will warrant taking into account the dynamics of this process in space and time. In view of this, Barovik and Taranchuk (2010) used finite difference approximation method to study the mathematical modeling of running crown forest fires and observed that, to create fire extinction it is necessary to increase the moisture content. Perminov (2018) estimated the amount of carbon dioxide and carbon monoxide emissions at crown forest fires spread using method of finite volume to obtain discrete analogies.

This study is aimed at establishing an analytical solution capable of determining the behavioral nature of the transient medium temperature and oxygen concentration in atmospheric-forest fire. This will be achieved using direct integration and eigenfunction expansion technique.

Model Formulations

A mathematical model of Forest fire spread is formulated based on balance equations for energy and fuel, where the fuel loss due to combustion which corresponds to the fuel reaction rate. The respective equations governing forest fire propagation are:

$$\frac{\partial \varphi_s}{\partial t} = -k_1 \varphi_s e^{-\frac{E_1}{RT}} \tag{1}$$

$$\frac{\partial \varphi_m}{\partial t} = -k_2 \varphi_m T^{\frac{1}{2}} e^{-\frac{E_2}{RT}} \tag{2}$$

$$\rho_c \frac{\partial \varphi_c}{\partial t} = \alpha_c k_1 \rho_s \varphi_s e^{-\frac{E_1}{RT}} - \frac{M_c}{M_1} k_3 S_\sigma \rho_g \varphi_c C_{ox} e^{-\frac{E_3}{RT}} \tag{3}$$

$$\left.\begin{aligned} &\rho_g \left(\frac{\partial C_{ox}}{\partial t} + V \frac{\partial C_{ox}}{\partial x} \right) = \frac{\partial}{\partial x}\left(\rho_g D_T \frac{\partial C_{ox}}{\partial x} \right) - \frac{\alpha}{C_{pg} \Delta h}\left(C_{ox} - C_{ox_\infty} \right) - \\ &\left(1-\alpha_c\right) k_1 \rho_s \varphi_s C_{ox} e^{-\frac{E_1}{RT}} - k_2 \rho_m T^{\frac{1}{2}} \varphi_m C_{ox} e^{\frac{-E_2}{RT}} - k_3 S_\sigma \rho_g \left(1 + \frac{M_c}{M_1} C_{ox} \right) \varphi_c C_{ox} e^{\frac{-E_3}{RT}} \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} &\left(\phi \rho_g C_{pg} + \left(1-\phi\right) \sum_{i=1}^{s+m+c} \rho_i C_{pi} \varphi_i \right) \frac{\partial T}{\partial t} + \rho_g C_{pg} V \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(\lambda_T \frac{\partial T}{\partial x} \right) - \frac{\alpha}{\Delta h}\left(T - T_\infty \right) \\ &-4K_R \sigma T^4 - k_2 \rho_m q_2 T^{\frac{1}{2}} \varphi_m e^{\frac{-E_2}{RT}} + k_3 S_\sigma \rho_g q_3 \varphi_c C_{ox} e^{\frac{-E_3}{RT}} \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} &\varphi_s\left(x,0\right) = \varphi_{so},\ \varphi_m\left(x,0\right) = \varphi_{mo},\ \varphi_c\left(x,0\right) = \varphi_{co},\ C_{ox}\left(x,0\right) = C_{ox_o},\ C_{ox}\left(0,t\right) = C_{ox_\infty} \\ &C_{ox}\left(\mathrm{L},t\right) = C_{ox_\infty},\ T\left(x,0\right) = T_o,\ T\left(0,t\right) = T_\infty,\ T\left(\mathrm{L},t\right) = T_\infty \end{aligned}\right\} \tag{6}$$

Such that;
Equations (1), (2) and (3) are volume fractions of dry organic substance, moisture and coke respectively while, (4), (5) and (6) are oxygen concentration, energy balance equation and initial and boundary conditions respectively.
Where;

φ_s is the volume fraction of dry organic substance, φ_m is the volume fraction of moisture, φ_c is the volume fraction of coke, C_{ox} is the concentration of oxygen, T is the temperature (in Kelvin), t is the time, x is a coordinate in the system of coordinates connected with the center of an initial fire (distance), T_∞ is the unperturbed ambient temperature, $k_j,\ j=1,2,3$ are the pre- exponential factors of chemical reactions, $E_j,\ j=1,2,3$ are the activation energy of chemical reactions, C is the concentration, R is the universal gas constant, S_σ is the specific surface of the condensed product of pyrolysis (coke), V is the equilibrium wind velocity vector, λ_T is the turbulent thermal conductivity, C_{ox_∞} is the unperturbed density of concentration of oxygen, $P_i, i=(s,m,c)$ is the i^{-th} phase density, that is ρ_s is the density of dry organic substance, ρ_m is the density of moisture, ρ_c is the density of coke, ρ_g is the density of gas phase (a mix of gases), Δh is the crown height, M_c is the molecular mass of carbon, M_1 is the mass of combustible forest material (CFM), C_{pg} is the thermal capacity of a gas phase, $q_j, j=2,3$ defines heat effects of processes of evaporation of burning, D_T is the diffusion coefficient, α is the coefficient of heat exchange between the atmosphere and a forest canopy, α_c is the coke number of combustible forest material (CFM), K_R is the Stefan-BoltzMann constant, $C_{p_i}, i=(s,m,c)$ is the i^{-th} phase of thermal capacity, s is the dry organic substance, m is the moisture, c is the coke, ox is the oxygen (O_2).

Method of Solution

Non-dimensionalisation

Here, equation (1) – (6) were non-dimensionalized using the dimensionless variables

$$\left.\begin{aligned} &x'=\frac{x}{L},\quad t'=\frac{Ut}{L},\quad v'=\frac{v}{U},\quad \psi_1=\frac{\varphi_s}{\varphi_{so}},\quad \psi_2=\frac{\varphi_m}{\varphi_{mo}},\quad \psi_3=\frac{\varphi_c}{\varphi_{co}},\quad \phi=\frac{C_{ox}-C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}} \\ &\in=\frac{RT_0}{E},\quad \theta=\frac{E\left(T-T_o\right)}{RT_o^{\,2}},\quad f=\frac{E_1}{E_3},\quad r=\frac{E_2}{E_3} \end{aligned}\right\} \qquad (7)$$

and we obtained;

$$\left.\begin{aligned}&\frac{\partial\psi_1}{\partial t}=-a\psi_1 e^{\frac{f\theta}{1+\in\theta}}\\&\psi_1(x,0)=1\end{aligned}\right\}\tag{8}$$

$$\left.\begin{aligned}&\frac{\partial\psi_2}{\partial t}=-b\psi_2(1+\in\theta)^{\frac{1}{2}}e^{\frac{r\theta}{1+\in\theta}}\\&\psi_2(x,0)=1\end{aligned}\right\}\tag{9}$$

$$\left.\begin{aligned}&\frac{\partial\psi_3}{\partial t}=\beta\psi_1 e^{\frac{f\theta}{1+\in\theta}}-\gamma(\phi+q)\psi_3 e^{\frac{\theta}{1+\in\theta}}\\&\psi_3(x,0)=1\end{aligned}\right\}\tag{10}$$

$$\left.\begin{aligned}&\frac{\partial\phi}{\partial t}+V\frac{\partial\phi}{\partial x}=\frac{\partial}{\partial x}\left(D_1\frac{\partial\phi}{\partial x}\right)-\beta_1\phi-\beta_2\psi_1(\phi+q)e^{\frac{f\theta}{1+\in\theta}}\\&-\beta_3(1+\in\theta)^{\frac{1}{2}}\psi_2(\phi+q)e^{\frac{r\theta}{1+\in\theta}}-\beta_4\psi_3(\phi+p)(\phi+q)e^{\frac{\theta}{1+\in\theta}}\\&\phi(x,0)=1,\quad\phi(0,t)=0,\quad\phi(1,t)=0\end{aligned}\right\}\tag{11}$$

$$\left.\begin{aligned}&\frac{\partial\theta}{\partial t}+V\frac{\partial\theta}{\partial x}=\frac{\partial}{\partial x}\left(\lambda_1\frac{\partial\theta}{\partial x}\right)-\alpha_1(\theta+\gamma_1)-R_a(1+4\in\theta)-\delta\psi_2(1+\in\theta)^{\frac{1}{2}}e^{\frac{r\theta}{1+\in\theta}}\\&+\delta_1\psi_3(\phi+q)e^{\frac{\theta}{1+\in\theta}}\\&\theta(x,0)=0,\quad\theta(0,t)=\sigma_1,\quad\theta(1,t)=\sigma_1\end{aligned}\right\}\tag{12}$$

where;

$$a=\frac{k_1Le^{\frac{-fE_1}{RT_o}}}{U},\quad b=\frac{k_2T_o^{\frac{1}{2}}Le^{\frac{-rE_3}{RT_0}}}{U},\ \beta=\frac{\alpha_c k_1\rho_s\varphi_{so}Le^{\frac{-fE_3}{RT_o}}}{U\rho_c\varphi_{co}},\ \gamma=\frac{M_ck_3S_\sigma\rho_gL}{M_1U\rho_c}\left(C_{ox_o}-C_{ox_\infty}\right)e^{\frac{-E_3}{RT_o}},$$

$$q=\frac{C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}},\ D_1=\frac{D_T}{LU}=\frac{1}{P_{em}},\ \beta_1=\frac{\alpha L}{C_{pg}\Delta hU},\ \beta_2=\frac{(1-\alpha_c)k_1\rho_s\varphi_{so}L}{\rho_gU}e^{\frac{-fE_3}{RT_o}},$$

$$\beta_3=\frac{k_2\rho_mT_o^{\frac{1}{2}}\varphi_{mo}Le^{\frac{-rE_3}{RT_o}}}{\rho_gU},\ \beta_4=\frac{k_3S_\sigma\rho_g\dfrac{M_c}{M_1}\left[C_{ox_o}-C_{ox_\infty}\right]L\varphi_{co}e^{\frac{-E_3}{RT_o}}}{\rho_gU},\ p=\frac{\dfrac{M_1}{M_c}+C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}},$$

$$\lambda_1 = \frac{\lambda_T}{L\rho_g C_{pg} U} = \frac{1}{P_e},\ \alpha_1 = \frac{\alpha L}{\rho_g C_{pg} U},\ R_a = \frac{4K_R \sigma L T_o^{\ 3}}{\rho_g C_{pg} \in U},\ \delta = \frac{k_2 \rho_m q_2 T_o^{\frac{1}{2}} L\varphi_{mo} e^{\frac{-rE_3}{RT_o}}}{\rho_g C_{pg} \in T_o U},$$

$$\delta_1 = \frac{k_3 S_\sigma \rho_g q_3 \varphi_{co} L\left(C_{ox_o} - C_{ox_\infty}\right)}{\rho_g C_{pg} \in T_o U} e^{\frac{-E_3}{RT_o}},\ \gamma_1 = \frac{T_o - T_\infty}{\in T_o}.$$

Analytical Solution

By the application of direct integration and eigenfunction expansion techniques, the analytical solution of equations (8)-(12) were obtained as follows:

$$\psi_1(x,t) = 1 + v\left(A_3 \sum_{n=1}^{\infty} A_2 \sin n\pi x - a_6 \left(\begin{array}{l} t + f(e-2). \\ \left(\sigma_1 t + \sum_{n=1}^{\infty} A_1 \left(t + \dfrac{e^{-c_2 t}}{c_2} \right) \sin n\pi x \right) \end{array} \right) \right) \tag{13}$$

$$\psi_2(x,t) = 1 + v\left(\begin{array}{l} a_7 \sum_{n=1}^{\infty} \mathrm{A}_2 \sin n\pi x \left(A_8 + A_7 \sum_{n=1}^{\infty} A_1 \sin n\pi x \right) - \\ a_7 \left(\begin{array}{l} t + r(e-2)\left(\sigma_1 t + \sum_{n=1}^{\infty} A_1 \left(t + \dfrac{e^{-c_2 t}}{c_2} \right) \sin n\pi x \right) + \\ \dfrac{1}{2} \in \left(\sigma_1 t + \sum_{n=1}^{\infty} A_1 \left(t + \dfrac{e^{-c_2 t}}{c_2} \right) \sin n\pi x \right) + \\ \dfrac{1}{2} \in \left(r(e-2) \right) \left(\begin{array}{l} \sigma^2{}_1 t + 2\sigma_1 \sum_{n=1}^{\infty} A_1 \left(t + \dfrac{e^{-c_2 t}}{c_2} \right) \sin n\pi x \\ + \sum_{n=1}^{\infty} A_1 \sum_{n=1}^{\infty} A_1 \left(t + \dfrac{2}{c_2} e^{-c_2 t} - \dfrac{1}{2c_2} e^{-2c_2 t} \right) \sin^2 n\pi x \end{array} \right) \end{array} \right) \end{array} \right) \tag{14}$$

$$\psi_3(x,t) = 1 + v\left(\begin{array}{l} a_9 \left(A_9 t + A_{10} \sum_{n=1}^{\infty} A_1 \left(t + \dfrac{e^{-c_2 t}}{c_2} \right) \sin n\pi x \right) \\ -a_8 \left(\begin{array}{l} -A_{11} \sum_{n=1}^{\infty} A \dfrac{e^{-c_1 t}}{c_1} \sin n\pi x + A_{12} \sum_{n=1}^{\infty} A \sum_{n=1}^{\infty} A_1 \left(-\dfrac{e^{-c_1 t}}{c_1} + \dfrac{e^{-(c_1+c_2)t}}{(c_1+c_2)} \right) \sin^2 n\pi x \\ + A_{13} t + A_{14} \sum_{n=1}^{\infty} A_1 \left(t + \dfrac{e^{-c_2 t}}{c_2} \right) \sin n\pi x \end{array} \right) \\ -a_9 A_{10} \sum_{n=1}^{\infty} A_2 \sin n\pi x + a_8 \left(\begin{array}{l} A_{14} \sum_{n=1}^{\infty} A_2 \sin n\pi x - A_{11} \sum_{n=1}^{\infty} \dfrac{A}{c_1} \sin n\pi x \\ -A_{12} \sum_{n=1}^{\infty} A \sum_{n=1}^{\infty} A_1 A_{15} \sin^2 n\pi x \end{array} \right) \end{array} \right) \tag{15}$$

$$\phi(x,t)=\sum_{n=1}^{\infty}Ae^{-c_1t}\sin n\pi x+v\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}-2A_{35}\sum_{n=1}^{\infty}Ate^{-c_1t}-2A_{36}\sum_{n=1}^{\infty}A_1\left[\frac{1}{c_1}-\frac{e^{-c_2t}}{(c_1-c_2)}+\frac{c_2e^{-c_1t}}{c_1(c_1-c_2)}\right]- \\ 2A_{37}\sum_{n=1}^{\infty}A\sum_{n=1}^{\infty}A_1A_{43}\left[te^{-c_1t}+\frac{e^{-(c_1+c_2)\tau}}{c_2}-\frac{e^{-c_1t}}{c_2}\right]- \\ 2A_{34}A_{42}\left(1-e^{-c_1t}\right)+2A_{40}\sum_{n=1}^{\infty}A^2\sum_{n=1}^{\infty}A_{42}\left(e^{-2c_1t}-e^{-c_1t}\right) \\ -2A_{38}\left(\sum_{n=1}^{\infty}A\sum_{n=1}^{\infty}A^2{}_1\sum_{n=1}^{\infty}\left[te^{-c_1t}+\frac{2e^{-(c_1+c_2)t}}{c_2}-\frac{e^{-(c_1+2c_2)t}}{2c_2}-\frac{3e^{-c_1t}}{2c_2}\right]\right) \\ -2A_{39}\sum_{n=1}^{\infty}A^2{}_1\sum_{n=1}^{\infty}A_{43}\left[\begin{array}{l}\frac{1}{c_1}-\frac{2e^{-c_2t}}{(c_1-c_2)}+\frac{e^{-2c_2t}}{(c_1-2c_2)}- \\ A_{44}\left(\begin{array}{l}e^{-c_1t}(c_1-c_2)(c_1-2c_2)- \\ 2e^{-c_1t}c_1(c_1-2c_2)+e^{-c_1t}c_1(c_1-c_2)\end{array}\right)\end{array}\right] \\ -2A_{41}\sum_{n=1}^{\infty}A^2\sum_{n=1}^{\infty}A_1\sum_{n=1}^{\infty}\left[\frac{e^{-(2c_1+c_2)t}}{(c_1+c_2)}-\frac{e^{-2c_1t}}{c_1}+\frac{c_2e^{-c_1t}}{c_1(c_1+c_2)}\right]\end{array}\right)\sin n\pi x\right) \tag{16}$$

$$\left.\begin{array}{l}\theta(x,t)=\left(\sigma_1+\sum_{n=1}^{\infty}A_1\left[1-e^{-c_2t}\right]\sin n\pi x\right)+ \\ v\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}-2A_{57}\sum_{n=1}^{\infty}A_1\left[\frac{1}{c_2}-te^{-c_2t}-\frac{e^{-c_2t}}{c_2}\right]- \\ 2\sum_{n=1}^{\infty}A^2{}_1\sum_{n=1}^{\infty}A_{51}\left[\frac{1}{c_2}-2te^{-c_2t}-\frac{e^{-2c_2t}}{c_2}\right] \\ +2A_{52}\sum_{n=1}^{\infty}A\left[\frac{e^{-c_1t}}{(c_2-c_1)}-\frac{e^{-c_2t}}{(c_2-c_1)}\right]-\frac{2A_{56}}{c_2}\left[1-e^{-c_2t}\right] \\ +2\sum_{n=1}^{\infty}A\sum_{n=1}^{\infty}A_1A_{53}\left[\frac{e^{-c_1t}}{(c_2-c_1)}+\frac{e^{-(c_2+c_1)t}}{c_1}-\frac{c_2e^{-c_2t}}{c_1(c_2-c_1)}\right]\end{array}\right)\sin n\pi x\right)\end{array}\right\} \tag{17}$$

Where;

$$A=\frac{2\left[1-(-1)^n\right]}{n\pi},\; A_1=\frac{2b_2\left[(-1)^n-1\right]}{n\pi c_2},\; A_2=\frac{A_1}{c_2},\; A_3=a_6f(e-2),\; A_4=r(e-2),\; A_5=\frac{1}{2}\in,$$

$$A_6 = \in\left(r\left(e-2\right)\right)\sigma_1,\ A_7 = \frac{3\in\left(r\left(e-2\right)\right)}{4},\ A_8 = \left(A_4 + A_5 + A_6\right),\ A_9 = \left(1 + f\left(e-2\right)\sigma_1\right),$$

$$A_{10} = f\left(e-2\right),\ A_{11} = \left(1+\left(e-2\right)\sigma_1\right),\ A_{12} = \left(e-2\right),\ A_{13} = \left(1+\left(e-2\right)\sigma_1\right)q,\ A_{14} = \left(e-2\right)q,$$

$$A_{15} = \frac{c_2}{c_1\left(c_1+c_2\right)},\ A_{16} = \left(1+f\left(e-2\right)\sigma_1\right)q,\ A_{17} = f\left(e-2\right)q,\ A_{18} = \left(1+r\left(e-2\right)\sigma_1\right),$$

$$A_{19} = r\left(e-2\right),\ A_{20} = \left(1+r\left(e-2\right)\sigma_1\right)q,\ A_{21} = r\left(e-2\right)q,\ A_{22} = \frac{1}{2}\in\left(\left(1+r\left(e-2\right)\sigma_1\right)\sigma_1\right),$$

$$A_{23} = \frac{1}{2}\in r\left(e-2\right)\sigma_1,\ A_{24} = \frac{1}{2}\in\left(\left(1+r\left(e-2\right)\sigma_1\right)\sigma_1 q\right),\ A_{25} = \frac{1}{2}\in r\left(e-2\right)\sigma_1 q,$$

$$A_{26} = \frac{1}{2}\in\left(1+r\left(e-2\right)\sigma_1\right),\ A_{27} = \frac{1}{2}\in r\left(e-2\right),\ A_{28} = \frac{1}{2}\in\left(1+r\left(e-2\right)\sigma_1\right)q,$$

$$A_{29} = \frac{1}{2}\in r\left(e-2\right)q,\ A_{30} = \left(p+q\right)\left(1+\left(e-2\right)\sigma_1\right),\ A_{31} = \left(p+q\right)\left(e-2\right),$$

$$A_{32} = \left(pq\right)\left(1+\left(e-2\right)\sigma_1\right),\ A_{33} = \left(pq\right)\left(e-2\right),\ A_{34} = \left(a_1A_{16} + a_2A_{20} + a_2A_{24} + a_3A_{32}\right),$$

$$A_{35} = \frac{1}{2}\left(a_1A_9 + a_2A_{18} + a_2A_{22} + a_3A_{30}\right),\ A_{36} = \frac{1}{2}\left(a_1A_{17} + a_2A_{21} + a_2A_{25} + a_2A_{28} + a_3A_{33}\right),$$

$$A_{37} = \frac{2}{3}\left(a_1A_{10} + a_2A_{19} + a_2A_{23} + a_2A_{26} + a_3A_{31}\right),\ A_{38} = \frac{3}{8}a_2A_{27},\ A_{39} = \frac{2}{3}a_2A_{29},$$

$$A_{40} = \frac{2}{3}a_3A_{11},\ A_{41} = \frac{3}{8}a_3A_{12},\ A_{42} = \left[\frac{1-\left(-1\right)^n}{n\pi c_1}\right],\ A_{43} = \left[\frac{1-\left(-1\right)^n}{n\pi}\right],\ A_{44} = \frac{1}{c_1\left(c_1-c_2\right)\left(c_1-2c_2\right)},$$

$$A_{45} = a_4A_{18}A_{43},\ A_{46} = \frac{a_4A_{19}}{2},\ A_{47} = a_4A_{22}A_{43},\ A_{48} = \frac{a_4A_{26}}{2},\ A_{49} = a_4A_{27}\sigma^2{}_1A_{43},\ A_{50} = a_4A_{27}\sigma_1,$$

$$A_{51} = \frac{2}{3}a_4A_{43},\ A_{52} = \frac{a_5A_{11}}{2},\ A_{53} = \frac{2a_5A_{12}A_{43}}{3},\ A_{54} = a_5A_{13}A_{43},\ A_{55} = \frac{a_5A_{14}}{2},$$

$$A_{56} = \left(A_{45} + A_{47} + A_{49} - A_{54}\right),\ A_{57} = \left(A_{46} + A_{48} + A_{50} + A_{55}\right),\ b_1 = \left(4R_a\in + \alpha_1\right),$$

$$b_2 = \left(\sigma_1\left(4R_a\in + \alpha_1\right) + \left(R_a + \alpha_1\gamma_1\right)\right),\ c_1 = \left(\beta_1 + D_1\left(n\pi\right)^2\right),\ c_2 = \left(b_1 + \lambda_1\left(n\pi\right)^2\right).$$

Equations (13)-(17) were computed using Maple 17.

Results and Discussion

To conclude this analysis, we examine the effect of activation energy parameter $\in$ on the transient state medium temperature $\theta(x,t)$ and oxygen concentration $\phi(x,t)$. Analytical solution given by equation (12)-(16), is computed using computer symbolic algebraic package MAPLE 17. The numerical result obtained from the method are shown in Figures 1 and 2 respectively.

Figure 1 depicts the graph of temperature $\theta(x,t)$ against distance x and time t for different values of dimensionless activation energy number $\in$. It is observed that the temperature decreases with time and decreases and later increases along distance but minimum medium temperature decreases as the activation energy number increases.

Figure 2 depicts the graph of oxygen concentration $\phi(x,t)$ against distance x and time t for different values of dimensionless activation energy number $\in$. It is observed that the oxygen concentration $\phi(x,t)$ increases and later become steady with time and oscillates along distance but maximum oxygen concentration increases as the activation energy number increases.

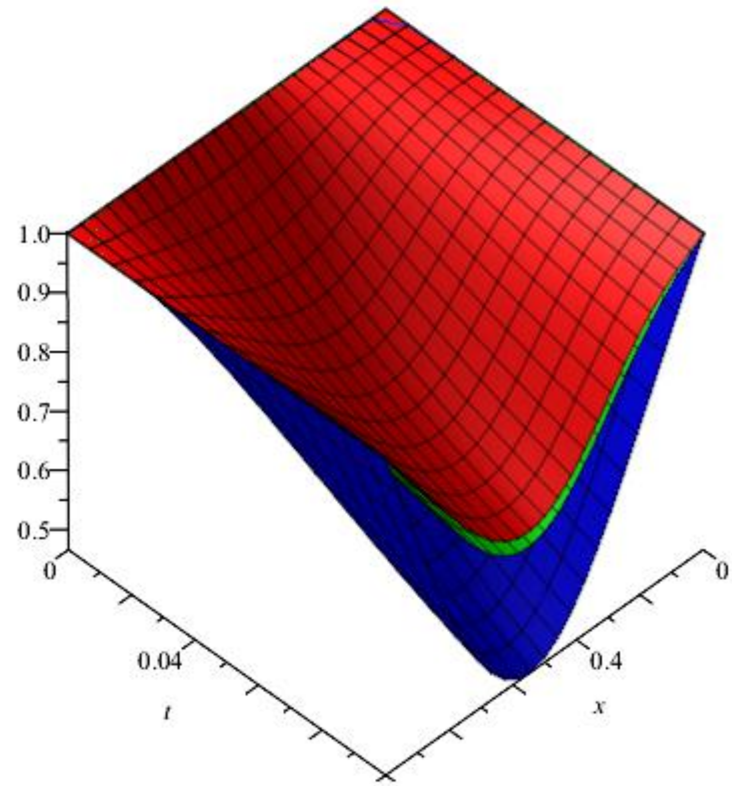

Figure 4.1: Graph of temperature $\theta(x,t)$ against distance x and time t for different values of dimensionless activation energy number $\in$. $(\in)=0.01(\text{Re}\,ad),(\in)=0.1(Green)\;and\;(\in)=1(Blue)$

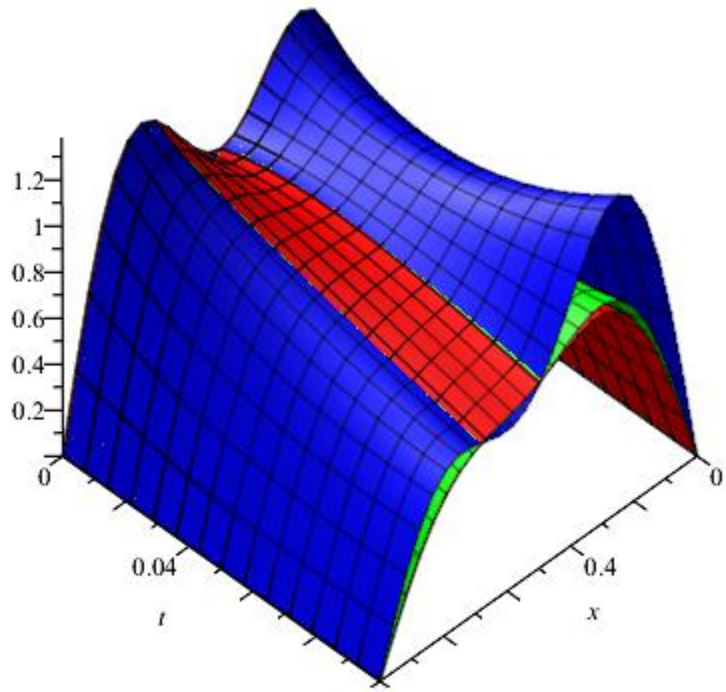

Figure 2: Graph of oxygen concentration $\phi(x,t)$ against distance x and time t for different values of dimensionless activation energy number $\in$. $(\in)=0.01(\mathrm{Re}\,ad),(\in)=0.1(Green)\ and\ (\in)=1(Blue)$

It is very crucial to point out that the effects observed in figures 1 and 2 are important for fire safety precautions.

Conclusion

For a high activation energy situation (i.e., as $\in \longrightarrow \mathbf{O}$), we have solved the equations governing the fire spread behavior via direct integration and eigenfunction expansion technique. From the result obtained, we can conclude that, Activation energy number reduced the medium temperature and enhanced the oxygen concentration.

These results obtained are expected to guide fire services to forecast the danger rating of forest fire spread behavior and to determine specific weather elements relating to forest fire propagation.

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