MATHEMATICAL MODELLING OF HEAT TRANSFER IN MICROCHANNELS

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Abstract
Plug flow can significantly enhance heat transfer in micro-channels as compared to single-phase flow. We investigate the impact of Peclet number and heat transfer coefficient on heat transfer in a liquid plug, solve the incompressible Newtonian fluid with constant properties and heat source, and solve the energy conservation equation by introducing a new space variable to provide an analytical solution to the two-dimensional heat transfer in the liquid plug. The governing equation was analytically handled utilizing Eigenfunction expansion techniques, which produced graphical summaries of the system responses on heat transfer, as well as the effect of Peclet number and heat transfer coefficient. The findings showed that as the Peclet number rises, so does the maximum temperature, and the temperature rises over time.

Introduction
High-powered electronic devices require very effective cooling to prevent overheating due to increased heat flux from their continued miniaturization. Microchannel heat sinks with high surface and volumetric heat transfer rates are used to accomplish this. Internal recirculation between the two phases promotes heat transfer by disturbing the intrinsic laminar flow within the microchannels. Self-fabricated water-based ferrofluid plugs make up the dispersed phase, while silicone oil makes up the continuous phase. The magnetic plugs of fluid are manipulated by an external magnetic field, causing more disturbance of the laminar flow than non-magnetic two-phase flow. Also, the ferrofluid plugs allow for easy separation of the two phases for pumping. Experimental results show that microchannel heat transfer using ferrofluid plugs is superior to that using deionized water as the dispersed phase for two-phase liquid-liquid plug flow and demonstrates that cooling performance is further enhanced by the application of an external magnetic field, inducing mixing within the flow (Gui et al, 2018).

Nguyen, (2016), Microchannel heat exchanger simulation provides a means to obtain design predictions at low cost and quick turnaround. Key output parameters such as overall pressure drop, heat exchanger effectiveness, and heat exchanger output can be calculated fast for many different designs using computational fluid dynamics to solve the conjugate heat transfer problem. The problem can be reduced in most microchannel heat exchangers by solving each channel flow independently. By substituting plug flow and Poiseuille flow models for the individual channels, computational efficiency can be improved, presenting the coupling of plug flow and Poiseuille flow models with ANSYS's computational fluid dynamics package, Fluent. The coupling algorithm is implemented between User Defined Functions and boundaries within Fluent's domains, eliminating the individual channel domain from the computational domain. Using the simplified model, full heat exchanger designs can be accurately represented with up to a 74% improvement in computational cost.

Tao-Wu *et. al* (2022) handle this kind of cooling problem under High-Temperature Conditions proposes liquid metal-based microchannel heat sinks. Using a numerical method, the flow and heat transfer performances of a liquid metal-based heat sink with different working fluid types, diverse microchannel cross-section shapes, and various inlet velocities were studied and discussed.

Model Formulation

To analyze the heat transfer in a liquid plug, we consider the energy conservation of the plug with a translating frame of reference following the plug. For an incompressible Newtonian fluid with constant properties and heat source, the energy conservation equation is

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_P}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+h\left(T-T_w\right) \tag{1}$$

The initial and boundary conditions are formulated as:

$$T\left(x,y,0\right)=T_0,\quad T\left(0,0,t\right)=T_w,\quad T\left(l,l,t\right)=T_w \tag{2}$$

Where:

T is the temperature, ρ is the fluid density, k is the thermal conductivity, C_P is the specific heat capacity, w is the width of the microchannel, v is the speed of the liquid plug, l is the length of the liquid plug, T_0 is the inlet temperature, T_w is the wall temperature, h is the heat transfer coefficient between the surface and the liquid plug, u is the velocity along the x-axis, v is the velocity along the y-axis, β is the dimensionless heat transfer coefficient.

Dimensional Analysis

Here, we non-dimensionalize equation (1) and (2.2), using the following dimensionless variables:

$$\theta=\frac{T-T_0}{T_w-T_0},\ x'=\frac{x}{l},\ y'=\frac{y}{l},\ u'=\frac{u}{v},\ v'=\frac{v}{v},\ t'=\frac{vt}{l} \tag{3}$$

and we obtain

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\rho e}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)+\sigma\left(\theta-1\right) \tag{4}$$

Together with initial and boundary conditions:

$$\theta\left(x,y,0\right)=0,\quad \theta\left(0,0,t\right)=1,\quad \theta\left(1,1,t\right)=1 \tag{5}$$

Method of Solution

In order to solve the dimensionless equations (4) and (5), we introduce a new space variable as;

$$z=x+y \tag{6}$$

Then, we obtain

$$\frac{\partial \theta}{\partial t}=D_\iota\frac{\partial^2 \theta}{\partial z^2}-U\frac{\partial \theta}{\partial z}+\sigma\left(\theta-1\right) \tag{7}$$

With the boundary and initial conditions

$$\theta\left(z,o\right)=0,\quad \theta\left(0,t\right)=1,\quad \theta\left(2,t\right)=1 \tag{8}$$

where

$D_\iota=\frac{2}{\rho e}$ and $U=u+v$

Next, we assume

$$\theta(z,t) = e^{\alpha z+\beta t}\phi(z,t) \tag{9}$$

Where α and β are constant parameters $\phi(z,t)$ is a new function

Then, we have

$$\frac{\partial\theta}{\partial t} = \beta e^{\alpha z+\beta t}\phi(z,t) + e^{\alpha z+\beta t}\frac{\partial}{\partial t}\phi(z,t) \tag{10}$$

$$\frac{\partial\theta}{\partial z} = \alpha e^{\alpha z+\beta t}\phi(z,t) + e^{\alpha z+\beta t}\frac{\partial}{\partial z}\phi(z,t) \tag{11}$$

$$\frac{\partial^2\theta}{\partial z^2} = \alpha^2 e^{\alpha z+\beta t}\phi(z,t) + 2\alpha e^{\alpha z+\beta t}\frac{\partial}{\partial z}\phi(z,t) + e^{\alpha z+\beta t}\frac{\partial^2}{\partial z^2}\phi(z,t) \tag{12}$$

Substituting equations (10), (11), (12) into equation (7) and (8) gives;

$$\frac{\partial\phi}{\partial t} = D_\iota\frac{\partial^2\phi}{\partial z^2} + (2D_\iota\alpha - U)\frac{\partial\phi}{\partial z} + \left(D_\iota\alpha^2 - U\alpha - \beta + \sigma\right)\phi(z,t) - \frac{\sigma}{e^{\alpha z+\beta t}} \tag{13}$$

To reduce equation (13) as a standard form of the dimensionless heat equation, the coefficient of the 2nd and 3rd terms of the RHS must equal zero. i.e.

$$2\alpha D_\iota - U = 0 \tag{14}$$

$$D_\iota\alpha^2 - U\alpha - \beta + \sigma = 0 \tag{15}$$

From equation (14) and (15), we obtain

$$\alpha = \frac{U}{2D_\iota}, \qquad \beta = \frac{U^2}{4D_\iota} - \frac{U^2}{2D_\iota} + \sigma \tag{16}$$

Now, the equations (7) and (8) change into the control equation: with initial and boundary conditions;

$$\frac{\partial\phi}{\partial t} = D_\iota\frac{\partial^2\phi}{\partial z^2} - \frac{\sigma}{e^{\alpha z+\beta t}} \tag{17}$$

with initial and boundary conditions;

$$\phi(z,0) = 0, \quad \phi(0,t) = e^{-\beta t}, \quad \phi(2,t) = e^{-(2\alpha+\beta t)} \tag{18}$$

Solving equation (17) and (18) using eigenfunction expansion method, we obtain

$$\phi(z,t) = \psi(z,t) + e^{-\beta t} + \frac{z}{2}\left(e^{-(2z+\beta t)} - e^{-\beta t}\right) \tag{19}$$

where

$$\psi(z,t) = \sum\psi_n(t)\sin\frac{n\pi z}{2} \tag{20}$$

$$\psi_n(t) = \frac{4\left(e^{-\beta t} - e^{-D_\iota\left(\frac{n\pi}{2}\right)t}\right)}{D_\iota(n\pi)^2 - 4\beta}\left[\frac{2\beta\left((-1)^n - 1\right)}{n\pi} + \frac{2n\pi\beta\left((-1)^n - 1\right)}{(2\alpha)^2\beta + (n\pi)^2}\right] + b_n e^{-D_\iota\left(\frac{n\pi}{2}\right)t} \tag{21}$$

The computation was done using MAPLE 17 version.

Results and Discussion

The equation describing the process of heat transfer in the microchannel is solved using the eigenfunction expansion method. The parameters of interest are Peclet number, velocity, and heat transfer coefficient. The solution obtained is computed using computer algebraic symbolic package MAPLE 17 version and the graphical representation is displayed in Figures 1 to 6 and discussed.

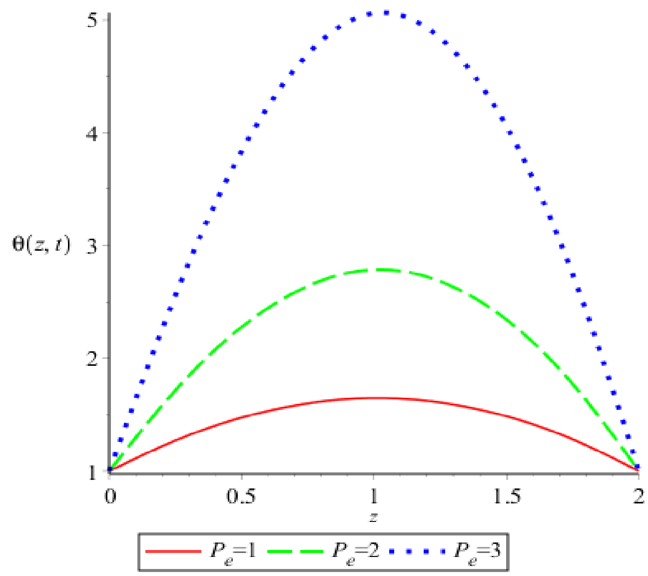

Figure 1: Variation of fluid temperature with Peclet number

Figure 1 shows the variation of fluid temperature with Peclet number. It is observed that the temperature increases and later decreases along with distance and maximum fluid temperature increases as the Peclet number increases.

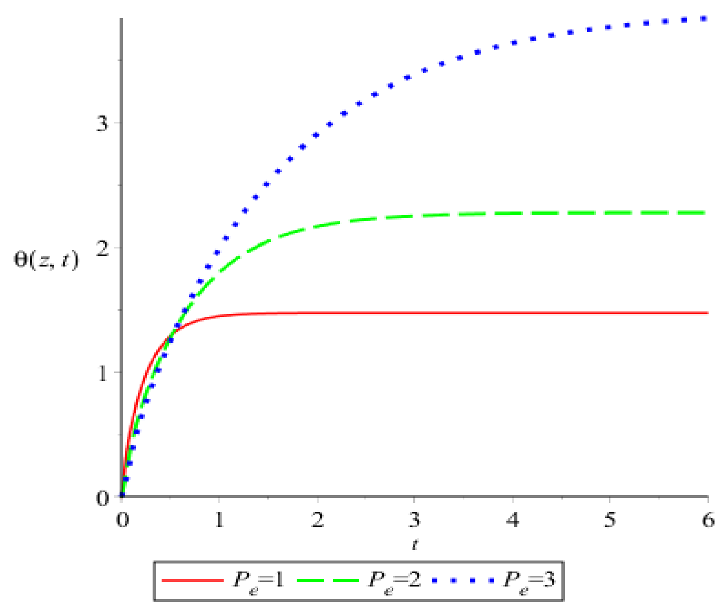

Figure 2: Variation of fluid temperature with Peclet number

Figure 2 shows the variation of fluid temperature with Peclet number. It is observed that the temperature increases with time and increases as the Peclet number increases.

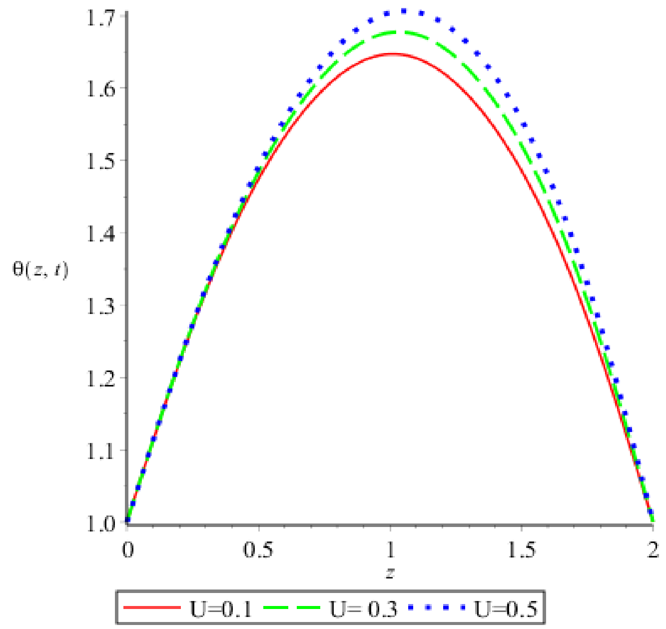

Figure 3: Variation of fluid temperature with velocity

Figure 3 shows the variation of fluid temperature with velocity. It is observed that the temperature increases and later decreases along with distance and maximum fluid temperature increases as velocity increases.

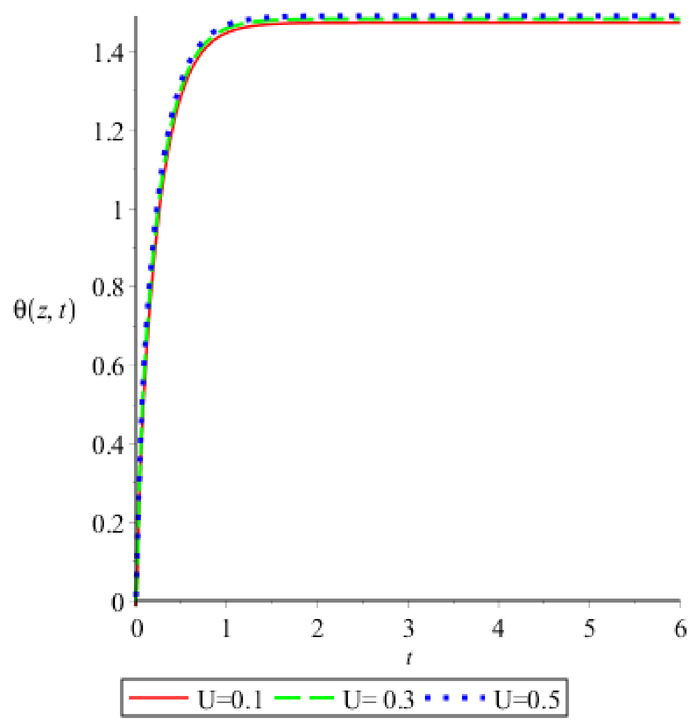

Figure 4: Variation of fluid temperature with velocity

Figure 4 shows the variation of fluid temperature with velocity. It is observed that the temperature increases with time and increases as velocity increases.

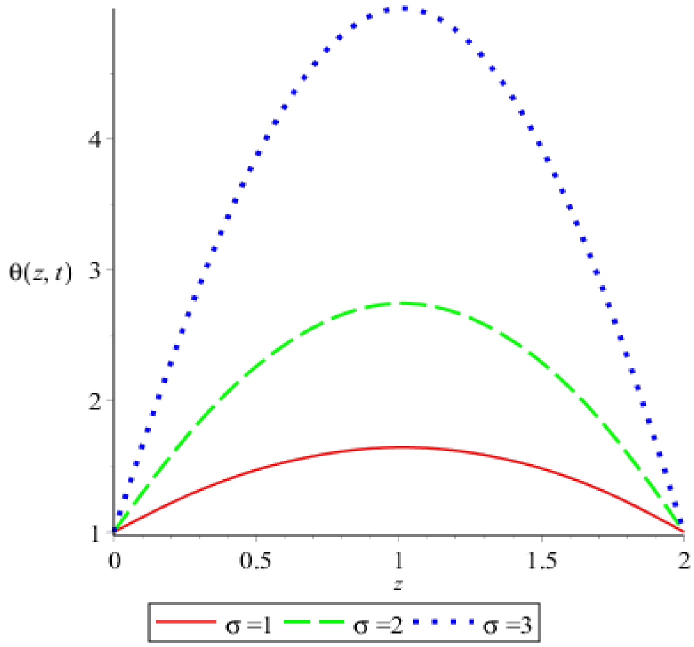

Figure 5: Variation of fluid temperature with heat transfer coefficient

Figure 5 shows the variation of fluid temperature with heat transfer coefficient. It is observed that the temperature increases and later decreases along with distance and maximum fluid temperature increases as the heat transfer coefficient increases.

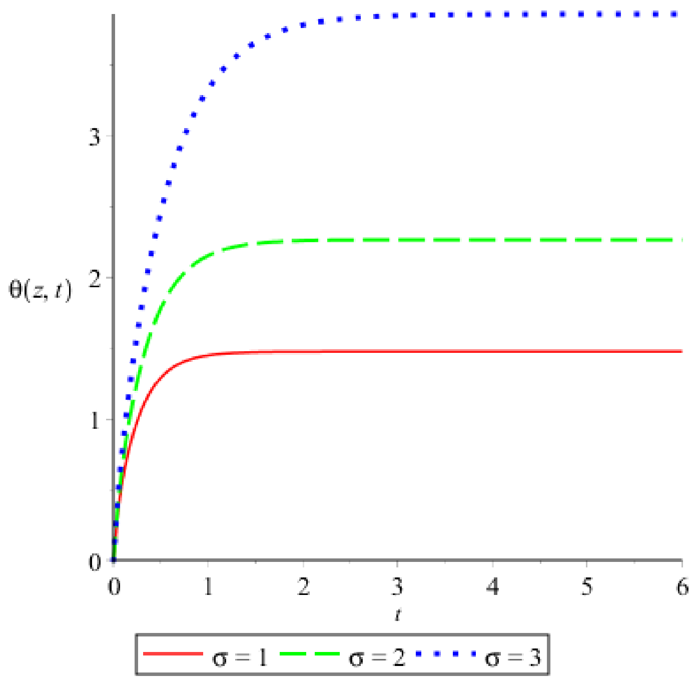

Figure 6: Variation of fluid temperature with heat transfer coefficient

Figure 6 shows the variation of fluid temperature with heat transfer coefficient. It is observed that the temperature increases with time and increases as the heat transfer coefficient increases.

Conclusion

In this paper, the heat transfers of plugs moving in micro-channels subjected to a constant surface temperature is investigated. The equation describing the heat transfer process in plugs moving in 2D micro-channels is solved using the eigenfunction expansion technique. The effects of the Peclet number, heat transfer coefficient, and velocity are studied. From the result obtained, we can conclude that.

(i) Peclet number enhanced the fluid temperature
(ii) Heat transfer coefficient enhanced the fluid temperature
(iii) Velocity of the fluid enhanced the fluid temperature
(iv) Fluid temperature is at maximum value when $z = 1$.

References

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