DERIVATION OF FORMULAS OF THE COMPONENTS OF KIFILIDEEN TRINOMIAL EXPANSION OF POSITIVE POWER OF N WITH OTHER DEVELOPMENTS

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Abstract

The sequence of the power combination of expansion of the Kifilideen trinomial theorem of positive power of n *is arranged in groups and patterns. Having a clearer view or picture of how the formulas of the components of the Kifilideen trinomial expansion were derived can lead to the discovery of formulating formulas for the series and sequences which follows the same pattern of progression as that of the trinomial expansion where series and sequence are not generated from trinomial expansion. Therefore, this study provides derivation for the formulas of the components of the Kifilideen trinomial expansion of positive power of* n *with other development. The research work also inaugurated Kifilideen theorem of matrix transformation of Newton's binomial theorem of positive and negative power of* n *and* $-n$ *where* a *and* b *are found in parts of the Newton's binomial expansion of the positive and negative power* n *and* $-n$*. The idea of series and sequence were employed in developing the formulas of the components of the Kifilideen trinomial expansion of positive power of* n*. The study indicates that there are sequences and series that can be developed which are not trinomial based but have the same pattern in term of progression as that of the latter. The Kifilideen theorem would ease the method of obtaining the power combination of a given term of Newton's binomial expansion in a systematic way.*

Keywords: Arithmetic sequence, Kifilideen general power combination formula, Kifilideen general group formula, Kifilideen general row column formula, Kifilideen matrix, Newton's binomial theorem, Series

Introduction

Sequence is the arrangement of natural number in certain pattern or form while series is the summing up the terms of the sequence (Fowler & Snapp, 2014). The word sequence come from the Greek word 'sequens' which means following. Sequences and series are very important in mathematics and also have many useful applications in areas such as finance, physics, statistics, biology, economics and medicine (Johnes, 2011; Brown, 2013). Arithmetic sequence and series are described as a pattern of number that has common and constant difference in between consecutive numbers where next term of the sequence is obtained by the addition or subtraction from the previous term (Talbert et al., 1995; Macrae et al., 2001; Fell et al., 2009). Arithmetic is one of the oldest branches of basic mathematics utilized to solve simple calculation. Carl Friedrich Gauss (1777 – 1855) a German Mathematician and Physicist born in Braunschweig, Germany; at his middle childhood age invented formula and also referred to as the father of Arithmetic sequence (Savita, 2014; Wittmann, 2020). He discovered formula to solve sum of arithmetic sequence at the age 7 (Richeson, 2019). His teacher named 'Herr Buttner' gave Carl's class task to sum 1 to 100which contain lengthy summation and that can take a lot of time to compute in order for the students to be busy for the entire day (Rice and Scott, 2005; Krance, 2019). As other students were busy carryout the task, talented Carl quickly discovered easiest method to solve it and provide solution to the task within a few moments which surprise the teacher and the entire class as he provided the correct answer as 5050 using a formula he discovered which was $\frac{N(N+1)}{2}$ (Pontes *et al.,* 2020). This indicates that discovery of new mathematical fact and pattern is opened to everyone either young or old. What matters are the interest, dedication, ability to put lots of effort, ability to develop oneself to do something differently in a unique way from the way it is usually done before and ability to think outside the box.

The Newton's binomial theorem of negative power of $-n$ is the theorem in expanding binomial expression (having two terms enclosed in a bracket) of negative power of $-n$ (Bird, 2003; Stroud & Booth, 2007). When the term of the expansion of Newton's binomial expression of power of n or $-n$ is given and the power combination of the term is to be found, using conventional method the general term is generated involving the unknown power combination (Bunday & Mulholland, 2014). Afterward, the general term generated is compare to the term given. In the process, scattered equations are produced which are then arranged and solved using substitution, elimination or crammer's rule. The introduction of the new Kifilideen theorem of matrix transformation of getting the power combination of a given term of Newton binomial expression of positive power of n or $-n$ provide systematic way of transforming the Newton's binomial expansion of positive power of n or $-n$ with the given term in which power combination is to be obtained into matrix directly and then the matrix is then solved to get the power combination. This Kifilideen theorem of matrix transformation had successfully been inaugurated for trinomial counterpart (Osanyinpeju, 2022a). This research work inaugurated Kifilideen theorem of matrix transformation of Newton's binomial theorem of positive and negative power of n and $-n$ where a and b are found in parts of the Newton's binomial expansion of positive and negative power n and $-n$. The Kifilideen theorem would ease the method of obtaining the power combination of a given term of a binomial expansion in a systematic way. The component parts of the power combination, kf of ${}^{n}_{kf}C$ of the binomial theorem are fully utilized during the matrix transformation process of the Newton's binomial expression of positive power of n or $-n$ and the given term unlike the conventional method where the power combination, r of ${}^{n}_{r}C$ is partial presented (Tuttuh & Adegoke, 2014; Osanyinpeju, 2021). Where k and f are the components parts of the fully presented power combination and r is the component of the partial presented power combination. Every component of a system has its uniqueness; neglecting one,results in hiding its potential, capability and value it can add to the system (Osanyinpeju, 2019; Osanyinpeju, 2020a).

The Kifilideen trinomial theorem of positive power of n is the theorem in expanding trinomial expression (having three terms enclosed in a bracket) of positive power of n (Osanyinpeju, 2020b). The sequences of the power combinations of the positive power of n of Kifilideen trinomial expansion are finite where for positive power of n the first and the last terms of the power combinations are $n00$ and $00n$ respectively (Osanyinpeju, 2020c). Matrix approach was used to arrange the power combinations of the Kifilideen trinomial expansion of positive power of n while series and sequence were utilized to generate formulas for the components of the Kifilideen trinomial theorem. The components of the Kifilideen trinomial theorem under study were power combination, term, group, row, column and position of the power combination. The acquisition of skill in generating and formulation of formulas for the components of the trinomial system required the foreknowledge of sequence and series. An important and integral part of the skill of sequence to implore in the formulation of formulas is to have the ability to recognize the pattern of progression of such components to formulate. The sequence of the power combinations of the Kifilideen Trinomial expansion of positive power of n in the Kifildeen matrix has some special arrangements where the power combinations decrease down the group in a constant magnitude of arithmetic progression of 90 and also decrease across the period in another constant magnitude of arithmetic progression of 81 in the Kifilideen matrix. The sequence of the power combination of expansion of the Kifilideen trinomial theorem of positive power of n is arranged in groups and patterns. Having a clearer view or picture of how the formulas of the components of the

Kifilideen trinomial expansion were derived can lead to the discovery of formulating formulas for the series and sequences which follows the same pattern of progression as that of the trinomial expansion where series and sequence are not generated from trinomial expansion. Therefore, this study provides derivation for the formulas of the components of the Kifilideen trinomial expansion of positive power of n with other development.

Materials and Methods

Derivation of the Kifilideen Power Combination Formula of Kifilideen Trinomial Theorem of Positive Power of n

Figure 1 shows the Kifilideen matrix which contains the array of the power combinations of the Kifilideen trinomial expansion of positive power of 2 in rows and columns. The groups 1, 2 and 3 have power combinations of terms 1 to 3, 4 to 5, and 6 respectively. Down the group (column), the power combination is arithmetically decreasing by 90 while across the row, the value of the power combination of the first member of one group and first member of the preceding group decrease arithmetically by 99. Furthermore, the values of the power combinations of the second member of one group and the second member of the preceding group decrease arithmetically by 99. The number of groups (columns) is $[n + 1 \ = 2 + 1 = 3]$ and number of rows is $[2n + 1 = 2 \times 2 + 1 = 5]$.

g_1	g_2	g_3
200		
110		
020	101	
	011	
		002

Figure 1: Kifilideen Matrix of Positive Power of 2

For group 1,

$$t = 1, C_p = 200 - 90 \times 0 - 99 \times 0 = 200 \quad (1)$$
$$t = 2, C_p = 200 - 90 \times 1 - 99 \times 0 = 110 \quad (2)$$
$$t = 3, C_p = 200 - 90 \times 2 - 99 \times 0 = 020 \quad (3)$$

For group 2

$$t = 4, C_p = 200 - 90 \times 0 - 99 \times 1 = 101 \quad (4)$$
$$t = 5, C_p = 200 - 90 \times 1 - 99 \times 1 = 011 \quad (5)$$

For group 3,

$$t = 6, C_p = 200 - 90 \times 0 - 99 \times 2 = 002 \quad (6)$$

Figure 2 indicates the Kifilideen matrix which contains the array of the power combinations of the Kifilideen trinomial expansion of positive power of 4 in rows and columns. The groups 1, 2, 3, 4 and 5 have power combinations of terms 1 to 5, 6 to 9, 10 to 12, 13 to 14 and 15 respectively. Down the group (column), the power combination is arithmetically decreasing by 90 while across the row, the values of the power combination of the first member of one group and first member of the preceding group decrease arithmetically by 99. Furthermore, the values of the power combinations of the second member of one group and the second member of the preceding group decrease arithmetically by 99 and so on.

g_1	g_2	g_3	g_4	g_5
400				
310				
220	301			
130	211			
040	121	202		
	031	112		
		022	103	
			013	
				004

Figure 2: Kifilideen Matrix of Positive Power of 4

For group 1,

$t = 1, C_p = 400 - 90 \times 0 - 99 \times 0 = 400$ (7)

$C_p = -90 \times 1 + 90 - [-81 + 90 \times 2] \times 0 + 400 = 400$ (8)

$C_p = -90 \times 1 + 81 \times 0 - 90 \times 0 + 490 = 400$ (9)

$C_p = -90 \times 1 + 81 \times 0 + 90 \times 0 + 490 = 400$ (10)

$t = 2, C_p = 400 - 90 \times 1 - 99 \times 0 = 310$ (11)

$C_p = -90 \times 2 + 90 - [-81 + 90 \times 2] \times 0 + 400 = 310$ (12)

$C_p = -90 \times 2 + 81 \times 0 - 90 \times 0 + 490 = 310$ (13)

$C_p = -90 \times 2 + 81 \times 0 + 90 \times 0 + 490 = 310$ (14)

$t = 3, C_p = 400 - 90 \times 2 - 99 \times 0 = 220$ (15)

$C_p = -90 \times 3 + 90 - [-81 + 90 \times 2] \times 0 + 400 = 220$ (16)

$C_p = -90 \times 3 + 81 \times 0 - 90 \times 0 + 490 = 220$ (17)

$C_p = -90 \times 3 + 81 \times 0 + 90 \times 0 + 490 = 220$ (18)

$t = 4, C_p = 400 - 90 \times 3 - 99 \times 0 = 130$ (19)

$C_p = -90 \times 4 + 90 - [-81 + 90 \times 2] \times 0 + 400 = 130$ (20)

$C_p = -90 \times 4 + 81 \times 0 - 90 \times 0 + 490 = 130$ (21)

$C_p = -90 \times 4 + 81 \times 0 + 90 \times 0 + 390 = 130$ (22)

$t = 5, C_p = 400 - 90 \times 4 - 99 \times 0 = 040$ (23)

$C_p = -90 \times 5 + 90 - [-81 + 90 \times 2] \times 0 + 400 = 040$ (24)

$C_p = -90 \times 5 + 81 \times 0 - 90 \times 0 + 490 = 040$ (25)

$C_p = -90 \times 5 + 81 \times 0 + 90 \times 0 + 490 = 040$ (26)

For group 2

$t = 6, C_p = 400 - 90 \times 0 - 99 \times 1 = 301$ (27)

$C_p = -90 \times 0 - [-81 + 90 \times 2] \times 1 + 400 = 301$ (28)

$C_p = -90 \times 6 + 81 \times 1 + 90 \times 3 + 490 = 301$ (29)

$t = 7, C_p = 400 - 90 \times 1 - 99 \times 1 = 211$ (30)

$C_p = -90 \times 1 - [-81 + 90 \times 2] \times 1 + 400 = 211$ (31)

$C_p = -90 \times 7 + 81 \times 1 + 90 \times 3 + 490 = 211$ (32)

$t = 8, C_p = 400 - 90 \times 2 - 99 \times 1 = 121$ (33)

$C_p = -90 \times 2 - [-81 + 90 \times 2] \times 1 + 400 = 121$ (34)

$$C_p = -90 \times 8 + 81\ \times 1 + 90 \times 3 + 490 = 121 \quad (35)$$

$$t = 9, C_p = 400 - 90 \times 3 - 99 \times 1 = 031 \quad (36)$$
$$C_p = -90 \times 3 - [-81 + 90 \times 2] \times 1 + 400 = 031 \quad (37)$$
$$C_p = -90 \times 9 + 81\ \times 1 + 90 \times 3 + 490 = 031 \quad (38)$$

For group 3,
$$t = 10, C_p = 400 - 90 \times 0 - 99 \times 2 = 202 \quad (39)$$
$$C_p = -90 \times 0 - [-81 + 90 \times 2] \times 2 + 400 = 202 \quad (40)$$
$$C_p = -90 \times 10 + 81\ \times 2 + 90 \times 5 + 490 = 202 \quad (41)$$

$$t = 11, C_p = 400 - 90 \times 1 - 99 \times 2 = 112 \quad (42)$$
$$C_p = -90 \times 1 - [-81 + 90 \times 2] \times 2 + 400 = 112 \quad (43)$$
$$C_p = -90 \times 11 + 81\ \times 2 + 90 \times 5 + 490 = 112 \quad (44)$$

$$t = 12, C_p = 400 - 90 \times 2 - 99 \times 2 = 022 \quad (45)$$
$$C_p = -90 \times 2 - [-81 + 90 \times 2] \times 2 + 400 = 022 \quad (46)$$
$$C_p = -90 \times 12 + 81\ \times 2 + 90 \times 5 + 490 = 022 \quad (47)$$

For group 4,
$$t = 13, C_p = 400 - 90 \times 0 - 99 \times 3 = 103 \quad (48)$$
$$C_p = -90 \times 0 - [-81 + 90 \times 2] \times 3 + 400 = 103 \quad (49)$$
$$C_p = -90 \times 13 + 81\ \times 3 + 90 \times 6 + 490 = 103 \quad (50)$$

$$t = 14, C_p = 400 - 90 \times 1 - 99 \times 3 = 013 \quad (51)$$
$$C_p = -90 \times 1 - [-81 + 90 \times 2] \times 3 + 400 = 013 \quad (52)$$
$$C_p = -90 \times 14 + 81\ \times 3 + 90 \times 6 + 490 = 013 \quad (53)$$

For group 5,
$$t = 15, C_p = 400 - 90 \times 0 - 99 \times 4 = 004 \quad (54)$$
$$C_p = -90 \times 0 - [-81 + 90 \times 2] \times 4 + 400 = 004 \quad (55)$$
$$C_p = -90 \times 15 + 81\ \times 4 + 90 \times 6 + 490 = 004 \quad (56)$$

From (10), (14), (18), (22), (26), (29), (32), (35), (38), (41), (44), (47), (50), (53) and (56) the coefficient of $[-90]$ in each equation is equivalent to the term of their power combination. Let the coefficient of 81 and 90 be a and $m$respectively in each equation of the power combination in (10), (14), (18), (22), (26), (29), (32), (35), (38), (41), (44), (47), (50) and (56). For 490 in the equations of the power combination (10), (14), (18), (22), (26), (29), (32), (35), (38), (41), (44), (47), (50), (53) and (56), the first digit 4 in the 490 represents the positive power of $4$of the Kifilideen trinomial theorem that generate the Kifilideen matrix. It can also be noted from (10), (14), (18), (22), (26), (29), (32), (35), (38), (41), (44), (47), (50, (53) and (56) that the coefficient of the [81] is equivalent to the last digitof the components of the power combination that produced it.

Generally,
$$C_p = -90t + 81a + 90m + n90 \quad (57)$$

The value of a is the equivalent to the last digit of the components of the power combination that produced it. From equations (10), (14), (18), (22), (26), (29), (32), (35), (38), (41), (44), (47), (50), (53) and (56); it is observed that in group 1 the values of a is the same for terms t = 1 to 4 and also the value of m is the same for terms t = 1 to 4. The same trend also follows in groups 2, 3, 4 and 5. Table 1 presents each group with the corresponding values of a and m for positive powers of 2, 3, 4, 5, 6, 7, 8, 9 and 10 of Kifilideen trinomial theorem. Note the value of a is the same for any term in any particular group likewise the value of m is the same for any term in any part particular group. In Table 1, it was indicated that the value of (a, m) for groups 1, 2, 3, 4 and 5 are $(0, 0)$, $(1, 3)$, $(2, 5)$, $(3, 6)$ and $(4, 6)$ respectively for positive power of n.

For positive power of 4: $[n = 4]$,

$g = 1;\ a = 0;\ m = 0 = 0$ (58)

$g = 2;\ a = 1;\ m = 0 + 3 = 3$ (59)

$g = 3;\ a = 2;\ m = 0 + 3 + 2 = 5$ (60)

$g = 4;\ a = 3;\ m = 0 + 3 + 2 + 1 = 6$ (61)

$g = 5;\ a = 4;\ m = 0 + 3 + 2 + 1 + 0 = 6$ (62)

For positive power of 6: $[n = 6]$,

$g = 1;\ a = 0;\ m = 0 = 0$ (63)

$g = 2;\ a = 1;\ m = 0 + 5 = 5$ (64)

$g = 3;\ a = 2;\ m = 0 + 5 + 4 = 9$ (65)

$g = 4;\ a = 3;\ m = 0 + 5 + 4 + 3 = 12$ (66)

$g = 5;\ a = 4;\ m = 0 + 5 + 4 + 3 + 2 = 14$ (67)

$g = 6;\ a = 5;\ m = 0 + 5 + 4 + 3 + 2 + 1 = 15$ (68)

$g = 7;\ a = 6;\ m = 0 + 5 + 4 + 3 + 2 + 1 + 0 = 15$ (69)

Table 1: Each group with the corresponding values of a and m for positive power of $2, 3, 4, 5, 6, 7, 8, 9,$ and 10 of Kifilideen trinomial theorem

		$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
$group(g)$	a	m	m	m	m	m	m	m	m	m
1	0	0	0	0	0	0	0	0	0	0
2	1	1	2	3	4	5	6	7	8	9
3	2	1	3	5	7	9	11	13	15	17
4	3		3	6	9	12	15	18	21	24
5	4			6	10	14	18	22	26	30
6	5				10	15	20	25	30	35
7	6					15	21	27	33	39
8	7						21	28	35	42
9	8							28	36	44
10	9								36	45
11	10									45

For positive power of n: $[n = n]$,

$g = 1;\quad a = 0;\quad m = 0$ (70)

$g = 2;\quad a = 1;\quad m = 0 + [n - 1]$ (71)

$g = 3;\quad a = 2;\quad m = 0 + [n - 1] + [n - 2]$ (72)

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$g = n + 1;\ a = n;\quad m = 0 + [n - 1] + [n - 2] + [n - 3] + \cdots + 2 + 1 + 0$ (73)

In (73) removing the first term [0] in the series of m; the number of term in the series of m then gives n. So, generally; for positive power of n,

When$a = n, m = [n-1] + [n-2] + [n-3] + \cdots + 2 + 1 + 0$ (74)
So, finding the sum of m; we have

First term = $[n-1]$; number of term $n = a$ and common difference = -1 (75)
According to McAskill et al. (2011) sum of A.P. is given as,
$S_n = \frac{n}{2}[2a + [n-1]d]$ (76)

Where S_n is the sum of the series of the n terms, a is the first term, n is the number of terms and d is the common differences. So we have:
$S_a = \frac{a}{2}[2[n-1] + [a-1] - 1]; \quad m = S_a$ (77)
$m = \frac{a}{2}[2n - a - 1]$ (78)
Also from (73), $g = a + 1$ (79)
Proved
Note, when $a = 0$, $m = 0$ (80)

Therefore, the Kifilideen general power combination formula for positive power of n of Kifilideen trinomial theorem is given as:
$C_p = -90t + 81a + 90m + n90$ (81)

Where value of a and m are determining using:
$a = g - 1$ and $m = \frac{a}{2}[2n - a - 1]$ (82)
The value of a is also equivalent to the last digit of the components of the power combination that produced it.

Derivation of Kifilideen general group formula of positive power of Kifilideen trinomial theorem
Figure 3 presents the Kifilideen matrix of positive power of 5 of Kifilideen trinomial theorem. The Kifilideen matrix has $[n + 1 = 5 + 1 = 6]$ six groups and $[2n + 1 = 2\times 5 + 1 = 11]$ eleven rows. The power combinations with colour red indicate the last power combination in each group. Table 2 presents the term, t^{th} of the last power combination of each group in the Kifilideen matrix of positive powers of 3, 4 and 5. Figure 3 and Table 2 indicate that the term of the last power combination of groups 1, 2, 3, 4, 5 and 6 in the Kifilideen matrix are the 6^{th}, 11^{th}, 15^{th}, 18^{th}, 20^{th} and $21^{st}$of Kifilideen trinomial theorem of positive power of 5.

g_1	g_2	g_3	g_4	g_5	g_6
500					
410					
320	401				
230	311				
140	221	302			
050	131	212			
	041	122	203		
		032	113		
			023	104	
				014	
					005

Figure 3: Kifilideen Matrix of Positive Power of 5

Table 2: The term, t^{th} of the last power combination of each group in the Kifilideen matrix of positive power of $3, 4$ and 5

$Groups\ [g]$	$n = 3$ t^{th}	$n = 4$ t^{th}	$n = 5$ t^{th}
1	$[4]^{th} = 4^{th}$	$[5]^{th} = 5^{th}$	$[6]^{th} = 6^{th}$
2	$[4+3]^{th} = 7^{th}$	$[5+4]^{th} = 9^{th}$	$[6+5]^{th} = 11^{th}$
3	$[4+3+2]^{th} = 9^{th}$	$[5+4+3]^{th} = 12^{th}$	$[6+5+4]^{th} = 15^{th}$
4	$[4+3+2+1]^{th} = 10^{th}$	$[5+4+3+2]^{th} = 14^{th}$	$[6+5+4+3]^{th} = 18^{th}$
5		$[5+4+3+2+1]^{th} = 15^{th}$	$[6+5+4+3+2]^{th} = 20^{th}$
6			$[6+5+4+3+2+1]^{th} = 21^{st}$

Table 3 presents the term, t^{th} of the last power combination of each group in the Kifilideen matrix of positive power of n.

Table 3: The term of the last power combination of each group in the Kifilideen matrix of positive power of n

$Groups\ [g]$	$n = n$ t^{th}
1	$[n+1]^{th}$
2	$[[n+1]+[n]]^{th}$
3	$[[n+1]+[n]+[n-1]]^{th}$
4	$[[n+1]+[n]+[n-1]+[n-2]]^{th}$
.	.
.	.
.	.
n	$[[n+1]+[n]+[n-1]+[n-2]+[n-3]+\cdots+[3]+[2]]^{th}$
$n+1$	$[[n+1]+[n]+[n-1]+[n-2]+[n-3]+\cdots+[3]+[2]+[1]]^{th}$

From Table 3, the term, t^{th} of the last power combination of each group in the Kifilideen matrix of positive power of n given as:

$g = 1, \quad t = [n+1]$ (83)

$g = 2, \quad t = [n+1]+[n]$ (84)

$g = 3, \quad t = [n+1]+[n]+[n-1]$ (85)

$g = 4, \quad t = [n+1]+[n]+[n-1]+[n-2]$ (86)

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$g = n, \quad t = [n+1]+[n]+[n-1]+[n-2]+[n-3]+\cdots+[3]+[2]$ (87)

$g = n+1, \quad t = [n+1]+[n]+[n-1]+[n-2]+[n-3]+\cdots+[3]+[2]+[1]$ (88)

Generally, for positive power of n; the term, $t^{th}$of the last power combination of group g is given by:

$g = n+1, \quad t = [n+1]+[n]+[n-1]+[n-2]+[n-3]+\cdots+[3]+[2]+[1]$ (89)

The series of the term is A.P. The first term of series of $t = [n+1]$, common difference= d $= -1$, the number of terms $= g = n+1$

According to Macrae *et al.* (2001) sum of A.P. is given as,

$S_n = \frac{n}{2}[2a + [n-1]d]$ (89)

Where S_n is the sum of the series of the n terms, a is the first term, n is the number of terms and d is the common differences. So, we have:

$S_g = \frac{g}{2}[2[n+1] + [g-1] - 1]; \quad t = S_g$ (90)

$t = \frac{g}{2}[2n - g + 3]$ (91)

Proved

Where t can be taken as any given term of a group $[g]$, n is the degree of the positive power of n and g is the group in which the given term belongs to. The (91) can be used to obtain the group of any given term of Kifilideen trinomial theorem of positive power of $n.$ If the value of group $[g]$ obtained when the term t^{th} given is inserted in (91) is whole number; that indicate that the power combination of the term is the last power combination of that group. For example, if the value of $g = 7$ that induces that the power combination of the term is the last power combination in that group 7. If the value of group $[g]$ obtained is decimal; that indicates that the power combination is not the last power combination in that group that it belongs to. To get the exact group of the term, the value of the whole number part of the value of the group obtained is round up to the next whole number. For example, if the value of $g = 6.10$ is obtained; so, the exact value of g is $g = 7$ that is; the term belongs to group 7. If the value of $g = 1.78$ is obtained; then the exact value of g is $g = 2$ that is; the term belongs to group 2.

Induction of the Kifilideen General Position Formula of Kifilideen Trinomial Theorem of Positive Power of n

Figure 4 indicates the Kifilideen matrix which contains the array of the power combinations of the Kifilideen trinomial expansion of positive power of 4 in rows and columns. In the Kifilideen matrix in Figure 4; the 1st, 2nd, 3rd, 4th and 5th positions, p for group 1 are 400, 310, 220, 130 and 040. For group 2; the 1st, 2nd, 3rd and 4th positions, p of the power combinations are 301, 211, 121 and 031. More so, for group 3; the 1st, 2nd and 3rd positions, p of the power combinations are 202, 112 and 022. Furthermore, for group 4; the 1st and 2nd positions, p of the power combinations are 103 and 013. Lastly, for group 5; the 1st position, p of the power combination is 004.

For group 1,

1st Position, $p = 1, \quad 400 = 400 - 90 \times 0 = 400 - 90 \times [1-1]$ (92)

2nd Position, $p = 2, \quad 310 = 400 - 90 \times 1 = 400 - 90 \times [2-1]$ (93)

3rd Position, $p = 3, \quad 220 = 400 - 90 \times 2 = 400 - 90 \times [3-1]$ (94)

4th Position, $p = 4, \quad 130 = 400 - 90 \times 3 = 400 - 90 \times [4-1]$ (95)

5th Position, $p = 5, \quad 040 = 400 - 90 \times 4 = 400 - 90 \times [5-1]$ (96)

p Position, $p = p, \quad R_{member} = F_{member} - 90[p-1] = F_{member} - 90p + 90$ (97)

Where R_{member} is the required power combination in which position is to be known in the group, F_{member} is the first power combination of the group in which the required power combination is found and p is the position of the required power combination in the group it belongs to.

g_1	g_2	g_3	g_4	g_5
400				
310				
220	301			
130	211			
040	121	202		
	031	112		
		022	103	
			013	
				004

Figure 4: Kifilideen Matrix of Positive Power of 4

For group 2,
The sequences of the power combination in the group 2 are 301, 211, 121 and 031.

First member = F_{member} = 301, common difference = $d = -90$ and p is the position of the term and Required member = R_{member}
According to Karris (2007),
$$T_n = a + [n-1]d \quad (98)$$

Where T_n is the term, a is the first term, n is the $\mathrm{n^{th}}$ term and d is the common difference.
So,
$$T_p = R_{member} = F_{member} + [p-1] \times -90 \quad (99)$$
$$R_{member} = F_{member} - 90p + 90 \quad (100)$$

For group 3,
1st Position, $p = 1$, $202 = 202 - 90 \times 0 = 202 - 90 \times [1-1]$ (101)
2nd Position, $p = 2$, $112 = 202 - 90 \times 1 = 202 - 90 \times [2-1]$ (102)
3rd Position, $p = 3$, $022 = 202 - 90 \times 2 = 202 - 90 \times [3-1]$ (103)
p Position, $p = p$, $R_{member} = F_{member} - 90[p-1] = F_{member} - 90p + 90$ (104)

Where R_{member} is the required power combination in which position is to be known in the group, F_{member} is the first power combination of the group in which the required power combination is found and p is the position of the required power combination in the group it belongs to.

For group 4,
1st Position, $p = 1$, $103 = 103 - 90 \times 0 = 103 - 90 \times [1-1]$ (105)
2nd Position, $p = 2$, $013 = 103 - 90 \times 1 = 103 - 90 \times [2-1]$ (106)
p Position, $p = p$, $R_{member} = F_{member} - 90[p-1] = F_{member} - 90p + 90$ (107)

Where R_{member} is the required power combination in which position is to be known in the group, F_{member} is the first power combination of the group in which the required power combination is found and p is the position of the required power combination in the group it belongs to.

For group 5,
1st Position, $p = 1$, $004 = 004 - 90 \times 0 = 004 - 90 \times [1-1]$ (108)
p Position, $p = p, R_{member} = F_{member} - 90[p-1] = F_{member} - 90p + 90$ (109)

WhereR_{member} is the required power combination in which position is to be known in the group, F_{member} is the first power combination of the group in which the required power combination is found and p is the position of the required power combination in the group it belongs to.

Generally, for any group in the Kifilideen matrix of positive power of n of Kifilideen trinomial theorem, the Kifilideen general position formula is derived as:

$$R_{member} = F_{member} - 90p + 90 \quad (110)$$

WhereR_{member} is the required power combination in which position is to be known in the group, F_{member} is the first power combination of the group in which the required power combination is found and p is the position of the required power combination in the group it belongs to.

Proved

Induction of the Kifilideen General Row Column Formula of Kifilideen Trinomial Theorem of Positive Power of n

Figure 5 indicates the Kifilideen matrix which contains the array of the power combinations of the Kifilideen trinomial expansion of positive power of 3 in rows and columns. The row and column $(r, c)$of the array of the power combinations 300, 210, 120, 030, 201, 111, 021, 102, 012 and 003 are (1, 1), (2, 1), (3,1), (4, 1), (3, 2), (4, 2), (5, 2), (5, 3), (6, 3) and (7, 4) respectively.

$$\left|\begin{array}{cccc}
g_1 & g_2 & g_3 & g_4 \\
C_{11} = 300 & & & \\
C_{21} = 210 & & & \\
C_{31} = 120 & C_{32} = 201 & & \\
C_{41} = 030 & C_{42} = 111 & & \\
 & C_{52} = 021 & C_{53} = 102 & \\
 & & C_{63} = 012 & \\
 & & & C_{74} = 003
\end{array}\right|$$

Figure 5: Kifilideen Matrix of Positive Power of 4

For group 1,

$rc = 11,\ C_{11} = 300 - 90 \times 0 - 99 \times 0 = 300$ (111)

$C_{11} = -90 \times 1 + 90 \times 0 - [-81 + 90 \times 2] \times 0 + 390 = 300$ (112)

$C_{11} = -90 \times 1 + 90 \times 0 + 81 \times 0 - 90 \times 0 + 390 = 300$ (113)

$C_{11} = -90 \times 1 + 81 \times 1 + 309 = 300$ (114)

$rc = 21,\ C_{21} = 300 - 90 \times 1 - 99 \times 0 = 210$ (115)

$C_{21} = -90 \times 2 + 90 \times 0 - [-81 + 90 \times 2] \times 0 + 390 = 210$ (116)

$C_{21} = -90 \times 2 + 81 \times 0 - 90 \times 0 + 390 = 210$ (117)

$C_{21} = -90 \times 2 + 81 \times 1 + 309 = 210$ (118)

$rc = 31,\ C_{31} = 300 - 90 \times 2 - 99 \times 0 = 120$ (119)

$C_{31} = -90 \times 3 + 90 \times 0 - [-81 + 90 \times 2] \times 0 + 390 = 120$ (120)

$C_{31} = -90 \times 3 + 90 \times 0 + 81 \times 0 - 90 \times 0 + 390 = 120$ (121)

$C_{31} = -90 \times 3 + 81 \times 1 + 309 = 120$ (122)

$rc = 41,\ C_{41} = 300 - 90 \times 3 - 99 \times 0 = 030$ (123)

$C_{41} = -90 \times 4 + 90 \times 0 - [-81 + 90 \times 2] \times 0 + 390 = 030$ (124)

$C_{41} = -90 \times 4 + 90 \times 0 + 81 \times 0 - 90 \times 0 + 390 = 030$ (125)

$C_{41} = -90 \times 4 + 81 \times 1 + 309 = 030$ (126)

For group 2

$rc = 32,\ C_{32} = 300 - 90 \times 0 - 99 \times 1 = 201$ (127)

$C_{32} = -90 \times 3 + 90 \times 2 - [-81 + 90 \times 2] \times 1 + 390 = 201$ (128)

$C_{32} = -90 \times 3 + 90 \times 2 + 81\ \times 1 - 90 \times 2 + 390 = 201$ (129)

$C_{32} = -90 \times 3 + 81\ \times 2 + 309 = 201$ (130)

$rc = 42,\ C_{42} = 300 - 90 \times 1 - 99 \times 1 = 111$ (131)

$C_{42} = -90 \times 4 + 90 \times 2 - [-81 + 90 \times 2] \times 1 + 390 = 111$ (132)

$C_{42} = -90 \times 4 + 90 \times 2 + 81\ \times 1 - 90 \times 2 + 390 = 111$ (133)

$C_{42} = -90 \times 4 + 81\ \times 2 + 309 = 111$ (134)

$rc = 52,\ C_{52} = 300 - 90 \times 2 - 99 \times 1 = 021$ (135)

$C_{52} = -90 \times 5 + 90 \times 2 - [-81 + 90 \times 2] \times 1 + 390 = 021$ (136)

$C_{52} = -90 \times 5 + 90 \times 2 + 81\ \times 1 - 90 \times 2 + 390 = 021$ (137)

$C_{52} = -90 \times 5 + 81\ \times 2 + 309 = 021$ (138)

For group 3,

$rc = 53,\ C_{53} = 300 - 90 \times 0 - 99 \times 2 = 102$ (139)

$C_{53} = -90 \times 5 + 90 \times 4 - [-81 + 90 \times 2] \times 2 + 390 = 102$ (140)

$C_{53} = -90 \times 5 + 90 \times 4 + 81\ \times 2 - 90 \times 4 + 390 = 102$ (141)

$C_{53} = -90 \times 5 + 81\ \times 3 + 309 = 102$ (142)

$rc = 63,\ C_{63} = 300 - 90 \times 1 - 99 \times 2 = 012$ (143)

$C_{63} = -90 \times 6 + 90 \times 4 - [-81 + 90 \times 2] \times 2 + 390 = 012$ (144)

$C_{63} = -90 \times 6 + 90 \times 4 + 81\ \times 2 - 90 \times 4 + 390 = 012$ (145)

$C_{63} = -90 \times 6 + 81\ \times 3 + 309 = 012$ (146)

For group 4,

$rc = 74,\ C_{74} = 300 - 90 \times 0 - 99 \times 3 = 003$ (147)

$C_{74} = -90 \times 7 + 90 \times 6 - [-81 + 90 \times 2] \times 3 + 390 = 003$ (148)

$C_{74} = -90 \times 7 + 90 \times 6 + 81\ \times 3 - 90 \times 6 + 390 = 003$ (149)

$C_{74} = -90 \times 7 + 81\ \times 4 + 309 = 003$ (150)

From (114), (118), (122), (126), (130), (134), (138), (142), (146) and (150); the coefficients of $[-90]$and [81] in each equation are equivalent to the row and column of their power combination respectively. For 309 in the equations of the power combination (114), (118), (122), (126), (130), (134), (138), (142), (146) and (150); the first digit 3 in the 390 represents the positive power of 3 of the trinomial theorem that generate the Kifilideen matrix.

So generally, for positive power of $n$of the Kifilideen trinomial theorem, we can say:

$C_{rc} = -90r + 81c + n09$ (151)

Where C_{rc} is the power combination in row r and column c; r is the row of the power combination; c is the column of the power combination and n is the degree of the positive power of n.

Proved

Table 5 presents the row r and column c down the group according to their position for positive power of 3. To determine the column of power combination we have:

$c\ =\ g$ and recall that $g\ =\ a\ +\ 1; c\ =\ g\ =\ a + 1$ (152)

Where c is the column of the power combination, g is the group of the power combination; a is the last digit of the components of the power combination that produce it.

Table 5: The row r and column c down the group according to their position for Kifilideen matrix of positive power of 3

$Position\ (p)$	$Group\ 1$		$Group\ 2$		$Group\ 3$		$Group\ 4$	
	$Row\ r$	Column c	$Row\ r$	Column c	$Row\ r$	Column c	$Row\ r$	Column c
1	$C_{11} \rightarrow 1$	1	$C_{32} \rightarrow 3$	2	$C_{53} \rightarrow 5$	3	$C_{74} \rightarrow 7$	4
2	$C_{21} \rightarrow 2$	1	$C_{42} \rightarrow 4$	2	$C_{63} \rightarrow 6$	3		
3	$C_{31} \rightarrow 3$	1	$C_{52} \rightarrow 5$	2				
4	$C_{41} \rightarrow 4$	1						

The Table 6 presents the row r and column $c$of the first position, p of group 1 to 4 for Kifilideen matrix of positive power of 3. The first row of each group or column is determined as shown:

For 1st position of the power combination from group 1 to 4;
From the Table 6, for the first position of power combination across the period; the first term of the row $r = 1$, common difference $= d = 3 - 1 = 2$ which can be noticed from Table 6 that the values of the row increases down arithmetically with 2. The Table 5 indicates the position of the row and column (r, c) of the first power combination for groups 1 to 4 with blue and red colours which are $(1,1)$, $(3,2)$, $(5,3)$, $(7,4)$ for groups 1, 2, 3 and 4 respectively which can also be seen in Table 6.

According to Bunday and Mulholland (2014),

$$T_n = a + [n-1]d \quad (153)$$

Therefore,

$$T_c = r = 1 + [c-1] \times 2 = 1 + 2[c-1] \quad (154)$$

For p position of the power combinations from group 1 to 4, we have:

$$r = p + 2[c-1] \quad (155)$$

Where r is the row of the power combination, c is the column of the power combination and p is the position of the power combination in the group.

Table 6: the row r and column c of the first position, p of group 1 to 4 of Kifilideen matrix of positive power of 3

Row r and column c of the first position, p of group 1 to 4	
Column c	Row r
1	1
2	3
3	5
4	7
c	r

Derivation of the Kifilideen Power Combination Formula of Newton's Binomial Theorem of Positive Power of n

The sequence of the power combinations of the Newton's binomial theorem of positive power of 5 is 50, 41, 32, 23, 14 and 05.
The first term $= 50$, $d = 41 - 50 = -9$ (156)

The array of the power combinations is Arithmetic progression; According to Bunday and Mulholland (2014),
$T_n = a + [n-1]d$ (157)

Where T_n is the term, a is the first term, d is the common difference and n is the n^{th} term
$T_t = C_p = 50 + [t-1] \times -9$ (158)

The sequence of the power combinations of the Newton's binomial theorem of positive power of $6$is 60, 51, 42, $33, 24$,15 and 06.
The first term= 60, $d = 51 - 60 = -9$ (159)
The array of the power combinations is Arithmetic progression, According to Talbert (1995),
$T_n = a + [n-1]d$ (160)

Where T_n is the term, a is the first term, d is the common difference and n is the n^{th} term
$T_t = C_p = 60 + [t-1] \times -9$ (161)

The sequence of the power combinations of the Newton's binomial theorem of positive power of $7$is 70, 61, 52, $43, 34$, 25, 16 and 07.
The first term= 70, $d = 61 - 70 = -9$ (162)
The array of the power combinations is Arithmetic progression, According to Stroud and Booth (2007),
$T_n = a + [n-1]d$ (163)

Where T_n is the term, a is the first term, d is the common difference and n is the n^{th} term
$T_t = C_p = 70 + [t-1] \times -9$ (164)

The sequence of the power combinations of the Newton's binomial theorem of positive power of n is $n0$, $[n-1]1$, $[n-2]2$, $[n-3]3, [n-4]4 , \ldots , 2[n-2]$, $1[n-1]$ and $0[n]$.
The first term= $n0$, $d = -9$ (165)
The array of the power combinations is Arithmetic progression, According to Macrae et al. (2001),
$T_n = a + [n-1]d$ (166)

Where T_n is the term, a is the first term, d is the common difference and n is the n^{th} term
$T_t = C_p = n0 + [t-1] \times -9$ (167)
$C_p = -9t + 9 + n0$ (168)
Proved

Induction of the Kifilideen Power combination Formula of Newton's Binomial Theorem of Negative Power of n
The sequence of the power combinations of the Newton's binomial theorem of negative power of -5 is -50, -61, -72,$-83, -94, \ldots$
The first term = -50, $d = -61 - -50 = -11$ (169)
The array of the power combinations is Arithmetic progression, According to Macrae et al. (2001),
$T_n = a + [n-1]d$ (170)

Where T_n is the term, a is the first term, d is the common difference and n is the n^{th} term
$T_t = C_p = -50 + [t-1] \times -11$ (171)

The sequence of the power combinations of the Newton's binomial theorem of negative power of – 6 is -60, -71, -82,$-93, 104$, -115,$-126, \ldots$,
The first term $= -60$, $d = -71 - 60 = -11$ (172)
The array of the power combinations is Arithmetic progression, According to Stroud and Booth (2007),
$T_n = a + [n - 1]d$ (173)

Where T_n is the term, a is the first term, d is the common difference and n is the n^{th} term
$T_t = C_p = -60 + [t - 1] \times -11$ (174)

The sequence of the power combinations of the Newton's binomial theorem of negative power of -7 is -70, -81, -92,-103,-114 , -125, -136,-147, $\ldots$
The first term $= -70$, $d = -81 - 70 = -11$ (175)
The array of the power combinations is Arithmetic progression, According to Talbert (1995) Talbert (1995),
$T_n = a + [n - 1]d$ (176)

Where T_n is the term, a is the first term, d is the common difference and n is the n^{th} term
$T_t = C_p = -70 + [t - 1] \times -11$ (177)

The sequence of the power combinations of the Newton's binomial theorem of negative power of $-n$ is $-n0$, $[-n - 1]1$, $[-n - 2]2$, $[-n - 3]3, [-n - 4]4, [-n - 5]5$, $[-n - 6]6$, $[-n - 7], \ldots$
The first term $= -n0$, $d = -11$ (178)
The array of the power combinations is Arithmetic progression, According to Tuttuh and Adegoke (2014),
$T_n = a + [n - 1]d$ (179)

Where T_n is the term, a is the first term, d is the common difference and n is the n^{th} term
$T_t = C_p = n0 + [t - 1] \times -11$ (180)
$C_p = -11t + 11 + n0$

Where n is negative degree of the negative power of the Newton binomial theorem.
Proved

Inauguration of Kifilideen Theorem of Matrix Transformation of Positive and Negative Power of n and $-n$ of Newton's Binomial Expression
If two variables a and b are found in each part of Newton's binomial expression of positive or negative power of n or $-n$ such as
$[ua^p b^q + va^r b^s]^n$ (181)

where n can be positive or negative and the power combination of any term in the Newton's binomial expansion of that kind of positive or negative power of the Newton's binomial expression is set as kf while the value of this term is designated as $wa^x b^y$.
Then, the Kifilideen matrix transformation of such positive or negative power of n or $-n$ of the Newton's binomial expression is of the form;

$$\begin{bmatrix} a \\ b \end{bmatrix}: \qquad \begin{bmatrix} p & r \\ q & s \end{bmatrix} \begin{bmatrix} k \\ f \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \tag{182}$$

Thus $k + f = n$ and where u, v and w are constants. k and f are the first and second digits of the component of the power combination of the term $wa^x b^y$. n is the degree of the Newton's binomial expression of positive or negative power.

More so,
$_{kf}^{n}Cu^{k}v^{f} = w$ (183)

Results

Implementation of Derivation of the Formulas of the Components of the Kifilideen Trinomial Theorem

If the power combination of a term in the Kifilideen trinomial expansion of positive power of n is 523; determine the following:

[i]the term that generate the power combination
[ii] the group and the position of the power combination in the Kifilideen matrix
[iii] the column and row the power combination belong to in the Kifilideen matrix

Solution

[A] Using Kifilideen general power combination formula for Kifilideen trinomial expansion of positive power of n, we have:
$C_p = -90t + 81a + 90m + n90$ (184)
The given power combination $= kif = 523$ (185)

Where value of a and m are determining using:
$a =$ the third digit of the components of the given power combination $= 3$ (186)
$n = k + i + f = 5 + 2 + 3 = 10$ (187)
$m = \frac{a}{2}[2n - a - 1] = \frac{3}{2}[2 \times 10 - 3 - 1] = 24$ (188)
$523 = -90t + 81 \times 3 + 90 \times 24 + 1090$ (189)
$t = 33^{rd}$ term (190)

[ii] $g = a + 1 = 3 + 1 = 4$ (191)
The power combination 523 is found in the 4^{th} group
Using the Kifilideen general position formula
$R_{member} = F_{member} - 90p + 90$ (192)
$F_{member} = 703$ (193)
$R_{member} = F_{member} - 90p + 90$ (194)
$523 = 703 - 90p + 90$ (195)
$p = 3^{rd}$ position in group 4 (196)

[iii] Using the Kifilideen general row column formula, we have
$c = a + 1 = 3 + 1 = 4^{th}$column (197)
$C_{rc} = -90r + 81c + n09$ (198)
$523 = -90r + 81 \times 4 + 1009$ (199)
$r = 9^{th}$ row (200)
Or
$r = p + 2[c - 1]$ (201)
$r = 3 + 2[4 - 1] = 9^{th}$ row (202)

Implementation of Kifilideen Theorem of Matrix Transformation of Newton's binomial Expression of Positive and Negative Power of n and $-n$

[B] Given that a term in the Newton's binomial expansion of $\left[\frac{b^4}{xa^2} - \frac{5a^7b^5}{7}\right]^n$ is $\frac{153600}{49}a^{24}b^{-10}$ where x is a constant value. Using Kifilideen theorem of matrix transformation of Newton's binomial expression of negative power of $-n$. Find

[i] the power combination
[ii] the degree of the power of the Newton's binomial expression
[iii] the t^{th} term
[iv] the value of x.

Solution

[i] Newton's binomial expression: $\left[\frac{1}{x}a^{-2}b^{4}+\frac{5}{7}a^{7}b^{5}\right]^{n}$
Power combination to be obtained: $k \quad f$
t[th] term of the power combination: $\frac{153600}{49}a^{24}b^{-10}$
Using the Kifilideen theorem of matrix transformation method, so
$\begin{bmatrix} a \\ b \end{bmatrix}$: $\begin{bmatrix} -2 & 7 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} k \\ f \end{bmatrix}=\begin{bmatrix} 24 \\ -10 \end{bmatrix}$ (203)
Also, $k+f=n$

Using Crammer's rule, so

$k=\frac{\Delta k}{\Delta}=\frac{\begin{vmatrix} 24 & 7 \\ -10 & 5 \end{vmatrix}}{\begin{vmatrix} -2 & 7 \\ 4 & 5 \end{vmatrix}}$ (204)

$k=\frac{190}{-38}$ (205)

$k=-5$ (206)

$f=\frac{\Delta f}{\Delta}=\frac{\begin{vmatrix} -2 & 24 \\ 4 & -10 \end{vmatrix}}{\begin{vmatrix} -2 & 7 \\ 4 & 5 \end{vmatrix}}$ (207)

$f=\frac{-76}{-38}$ (208)

$f=2$ (209)

So, the power combination $=kf=-52$ (210)
[ii] the positive power of n of the binomial expression $=n=k+f=-5+2=-3$ (211)
[iii] Using the Kifilideen general power combination formula of binomial expression of negative power of n

$C_p=-11t+11+n0$ (212)

$-52=-11t+11-30$ (213)

$t=3^{rd}$ term (214)

From the question, $u=\frac{1}{x}, v=\frac{5}{7}$ $and\ q=\frac{153600}{49}$ (215)
Using Kifilideen matrix transformation method,

${}^{n}_{kf}Cu^{k}v^{f}=\frac{153600}{49}$ (216)

${}^{-3}_{-52}C[\frac{1}{x}]^{-5}[\frac{5}{7}]^{2}=\frac{153600}{49}$ (217)

$6\times x^{5}\times\frac{25}{49}=\frac{153600}{49}$ (218)

$x^{5}=4^{5}$ (219)

$x=4$ (220)

[iii] Given that a term in the Newton's binomial expansion of $\left[\frac{3a^{4}}{b^{5}}+\frac{cb}{a^{3}}\right]^{n}$ is $1088640a^{-2}b^{-14}$ where c is a constant value. Using Kifilideen theorem of matrix transformation of Newton's binomial expression of power of n. Find
[i] the power combination
[ii] the degree of thepower of the Newton's binomial expression
[iii] the t^{th} term
[iv] the value of c.

Solution

[i] Newton's binomial expression: $[3a^4b^{-5} + ca^{-3}b]^n$

Power combination to be obtained: $k \quad f$

t^{th} term of the power combination: $1088640a^{-2}b^{-14}$

Using the Kifilideen theorem of matrix transformation method, so

$\begin{bmatrix} a \\ b \end{bmatrix}$: $\begin{bmatrix} 4 & -3 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} k \\ f \end{bmatrix} = \begin{bmatrix} -2 \\ -14 \end{bmatrix}$ (221)

Also, $k + f = n$

Using Crammer's rule, so

$k = \frac{\Delta k}{\Delta} = \frac{\begin{vmatrix} -2 & -3 \\ -14 & 1 \end{vmatrix}}{\begin{vmatrix} 4 & -3 \\ -5 & 1 \end{vmatrix}}$ (222)

$k = \frac{-44}{-11}$ (223)

$k = 4$ (224)

$f = \frac{\Delta f}{\Delta} = \frac{\begin{vmatrix} 4 & -2 \\ -5 & -14 \end{vmatrix}}{\begin{vmatrix} 4 & -3 \\ -5 & 1 \end{vmatrix}}$ (225)

$f = \frac{-66}{-11}$ (226)

$f = 6$ (227)

So, the power combination $= kif = 46$ (228)

[ii] the positive power of n of the binomial expression $= n = k + f = 4 + 6 = 10$ (229)

[iii] Using the Kifilideen general power combination formula of binomial expression of positive power of n

$C_p = -9t + 9 + n0$ (230)

$46 = -9t + 9 + 100$ (231)

$t = 7^{th}$term (232)

Or

$t = f + 1 = 6 + 1 = 7$ (233)

From the question, $u = 3, v = c \ and \ q = 1088640$ (234)

Using Kifilideen matrix transformation method,

${}^{n}_{kf}Cu^k v^f = 1088640$ (235)

${}^{10}_{46}C[3]^4[c]^6 = 1088640$ (236)

$210 \times 81 \times [c]^6 = 1088640$ (237)

$c = 2$ (238)

Conclusion

This study provides derivation for the formulas of the components of the Kifilideen trinomial expansion of positive power of n with other developments. The research work also inaugurated Kifilideen theorem of matrix transformation of Newton's binomial theorem of positive and negative power of n and $-n$ where a and b are found in parts of the Newton's binomial expansion of positive and negative power n and $-n$. The idea of series and sequence were employed in developing the formulas of the components of the Kifilideen trinomial expansion of positive power of n. The study indicates that there are sequences and series that can be developed which are not trinomial based but have the same pattern in term of progression as that of the latter. The Kifilideen theorem would ease the method of obtaining the power combination of a given term of Newton's binomial expansion in a systematic way.

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