POLYTECHNIC STUDENTS' UNDERSTANDING OF THE CONCEPT OF LIMITS IN A CALCULUS COURSE THROUGH CONSTRUCTIVIST APPROACH

MUHAMMAD ABDURRAHMAN SANI, PhD; MUKHTAR MUHAMMAD SANI; & FARUK ABDULLAHI
Department of Mathematics,
Waziri Umaru Federal Polytechnic Birnin Kebbi, Kebbi State, Nigeria
E-mail: abdurrahman@wufpbk.edu.ng

Abstract
Students learn in a variety of ways, with some instructors lecturing, others demonstrating/discussing, and some focusing on principles while others on applications. As a result, the level of understanding and prior preparation of a student's learning style is affected. Because learners develop their own knowledge or at least interpret it based on their perceived experiences, one of the most essential ways for teaching calculus is the constructive approach. In this study, two research questions were raised, and two null hypotheses were formulated to guide the study. A pre-test post-test quasi-experimental design was adopted, with one hundred (100) students randomly sampled from the population of all ND 2 students taking the course of Calculus for Science (STP 213). The research instruments used were self-developed pre-test and post-test that was validated by experts. The instruments were pilot tested and confirmed to be reliable using Cronbach's alpha (α=0.82) coefficient. The data were statistically analyzed using mean, SD, and independent sample t-test. The findings revealed that using a constructivist approach, polytechnic students' understanding of the concepts of limits can be improved.

Keywords: Calculus, constructivist approach, limits, polytechnic education, students centred learning

Introduction
Polytechnic education is an important part of higher education that prepares students in technical and vocational fields so as to be self-employed and as well create jobs for others after graduation. This is accomplished through the awarding of Certificates, National Diplomas, Higher National Diplomas, and Advanced Professional Diplomas that are relevant to the nation's diverse economy and industries' needs, expectations, and growth. Polytechnic graduates are required to be well-versed in the application of theoretical principles in order to address long-term social and economic issues (Muhammad *et al.*, 2020). In Nigeria and around the world, polytechnics arose from the necessity to scale up the production and transmission of technical education for long-term economic growth and development (Baba, 2021). It was founded in accordance with Nigeria's National Policy on Education to provide middle-quality technical knowledge and skills for the country's overall growth (Muhammad, Abdullah & Osman, 2021). Micheal and Iduma (2013), Uwaifo (2010) opined that polytechnics are expected to be key participants in Nigeria's economic diversification and long-term development. No one can deny that today's global economy is knowledge-driven, with science and technology playing a critical role. As a result, polytechnics play an essential role in many modern economies' technological growth (Kamoru, 2021). However, due to insufficient infrastructure, low finance, negative perceptions, and lack of commitment on the part of organizations on the side of polytechnic graduates through dissatisfaction and discrimination, its impact was little.

The concept of limits is a vital and fundamental concept in a calculus course, and successfully teaching it is not impossible provided that suitable techniques and instruments are used. In today's calculus instruction, the concept of limit occupies an ironic position because is the foundation of modern calculus as well as a prerequisite for courses such as measure theory, real analysis, and functional analysis (Liang, 2016). The concepts of functions and limits are frequently introduced to students as they begin to learn calculus. If mathematics majors do not understand the concept of limit, they will struggle to understand the concepts of continuity, uniform continuity, convergence, and derivative, and will be unprepared to tackle other analysis courses (Juter, 2007; Stewart, 2012). Lack of understanding of the concept of limit may not be a serious problem for non-mathematics majors in their courses pursuit but, it is of significant concern for mathematics majors. As real number L is called the limit of a function in a neighbourhood of a point x_0, if and only if $f(x) \to L$ as $x \to x_0$ normally written $\lim_{x \to 0} f(x) = L$.

Constructivist approach involves a process whereby learners construct their own knowledge or at least interpret it based on their perceptions of experiences (Sani, Garba & Abdullahi, 2014). Constructivist learning is one strategy that can enable all the learners to construct valid knowledge and also enable them to transmit it in different contexts. In calculus, constructivist pedagogy holds that rather than obtaining knowledge by passive observation of teachers' demonstrations in the classroom, students can construct knowledge by actively participating in calculus discussions and solving new or unexpected problems (Sani et al., 2014). The learner's position as an active participant in the learning process is emphasized in the constructivist learning approach. This requires students to become self-directed learners and uncover aspects of research, concepts, or ideas on their own, rather than simply watching or participating in the activity (Sani *et al.*, 2014).

According to Chukwuyenum (2013), Muhammad, Abdullah, and Osman (2020), constructivist learning has been adopted in Nigerian institutions as one of the nation's educational objectives to make students creative; nevertheless, it has not been directly integrated and has low implementation. A core objective in the development of polytechnic education is for students to be able to solve calculus problems using a student-centered method. According to studies, the content of limits is primarily taught through a teacher-centered method; nevertheless, what is required currently in terms of pedagogy is a teaching strategy that stresses students' participation in knowledge development rather than knowledge acquisition and transformation. According to Liang (2016), one of the students' sources of minimal knowledge of the core concepts of limits is a lack of instructional practices that allow students to communicate with one another and develop their own ideas. Furthermore, it is possible that some calculus teachers themselves do not fully comprehend or have misconceptions about the idea of limits (Bokhari & Yushau, 2006).

Research Objectives

The study intends to achieve the following objectives:

(i) Determine the effect of guided discovery approach in understanding the concepts of limits among polytechnic students.

(ii) Investigate the effect of guided discovery approach on gender in understanding the concepts of limits among polytechnic students.

Research Questions

In line with the objectives of the study, the following research questions were raised in this study:

(i) Does guided discovery approach affect polytechnic students' academic performance in understanding the concepts of limits?

(ii) How does guided discovery approach affect both male and female students in understanding the concepts of limits?

Research Hypotheses

From the research questions, the following null hypotheses were formulated and tested at 0.05 level of significance:

Ho_1: There is no significant difference between the performances of students taught limits using guided discovery approach and those taught using conventional approach.

Ho_2: There is no significant difference between the mean scores of male and female students in limits using guided discovery approach.

Methodology

A pre-test post-test quasi-experimental design was used for this study involving two groups (experimental and control). In this type of design, the groups were observed and analysed before and after being exposed to a treatment (Sani, 2017). This study took place in a polytechnic, targeting all National Diploma II (ND II) students offering the course of calculus for science (STP 213). Two ND II classes are randomly selected using simple balloting method as the study samples and a total of 100 students (56 and 44 for both experimental and control groups respectively) constituted the study's sample as shown in Table 1. This number of students is adequate for the collection and analysis of quantitative data (Sani, 2017).

Table 1: Samples selected for the study

S/N	Group	Program	Males	Females	Total
1	Experimental	ND 2 Regular program	31	25	56
2	Control	ND 2 Evening program	24	20	44
		Total	55	45	100

The research instruments used in this research developed by the researchers (pre-test & post-test) and were validated by experts. In order to consider the instruments valid, adjustments were made based on expert comments. The instruments were pilot tested, and Cronbach's alpha internal consistency revealed a reliability coefficient of 0.82. As a result, any inferences drawn from the reliability coefficient's result are valid (Muhammad *et al.*, 2021; Sani, 2017).

Results

Prior to the intervention process, the two groups were pre-tested to assess their level of homogeneity and understanding in the topic area. The groups were instructed separately and were given the same post-test after meeting once a week for a minimum of two hours for a period of seven weeks. Using SPSS software, the results from the pre-test and post-test were statistically analyzed to determine the effectiveness of the constructive approach among polytechnic students in the topic of limits. Using independent sample t-test statistic, Table 2 below compares the pre-test scores of both the experimental and control groups.

Table 2: T-test comparison for both experimental and control groups on pre-test

Group	N	$\overline{X}$	SD	SE_M	df	t-val.	p-value	Remark
Experimental	56	30.08	3.52	0.83	98	0.169	0.174	Not Sig.
Control	44	29.91	3.17	0.69				

Table 2 shows the pre-test independent sample t-test statistic for both the experimental and control groups. A significant level of alpha=0.05 was used, and a mean difference of 0.17 was obtained by comparing the pre-test scores of the experimental (M=30.08, SD=3.52) and control (M=29.91, SD=3.17) groups. At t(98)=0.169 and p>0.001, this difference was considered to be statistically non-significant. The Cohen's d for this test was 0.013, which indicates a very small effect size in the difference of mean (Cohen *et al.*, 2017). Using independent sample t-test statistic, Table 3 below compares the post-test scores of both the experimental and control groups.

Table 3: T-test comparison for both experimental and control groups on post-test

Group	N	$\overline{X}$	SD	SE_M	df	t-val.	p-value	Remark
Experimental	56	53.14	4.58	0.81	98	16.741	0.000	Sig.
Control	44	40.27	4.72	0.89				

Table 3 shows the post-test independent sample t-test statistic for both the experimental and control groups. A significant level of alpha=0.05 was used, and a mean difference of 12.87 was obtained by comparing the post-test scores of the experimental (M=53.14, SD=4.58) and control (M=40.27, SD=4.72) groups. At $t(98)=16.741$ and $p<0.001$, this difference was considered to be statistically significant. The Cohen's d for this test was 4.36, which indicates a large effect size in the difference of mean (Cohen *et al.*, 2017). Using independent sample t-test statistic, Table 4 below compares the post-test scores of both gender.

Table 4: T-test comparison for gender on post-test scores in limits

Gender	N	$\overline{X}$	SD	SE_M	df	t-val.	p-value	Remark
Male	55	50.62	4.09	0.93	98	14.925	0.000	Sig.
Female	45	39.77	3.83	0.85				

Table 4 shows the post-test independent sample t-test statistic for both genders. A significant level of alpha=0.05 was used, and a mean difference of 10.85 was obtained by comparing the post-test scores of both male (M=50.62, SD=4.09) and female (M=39.77, SD=3.83) gender. At $t(98)=14.925$ and $p<0.001$, this difference was considered to be statistically significant. The Cohen's d for this test was 3.04, which indicates a large effect size in the difference of mean (Cohen *et al.*, 2017).

Discussion

In a calculus course, students' understanding of the concept of limits has long been under-addressed; researchers discovered that first-year university students' knowledge and understanding is based on discrete facts and procedures in a variety of ways (Bokhari & Yushau, 2006). Students describe it as a non-passable border, while others see it as an estimated value reached through an evaluative process or by conceptualizing points on a graph moving closer to the limit (Liang, 2016) as well as have divergent ideas about limit, continuity, and differentiability. These divergent viewpoints could be a reflection of the informal mental models students have developed based on earlier experiences, such as non-mathematical understandings of limit, which frequently cause students to struggle with subsequent calculus concepts (Klymchuk, 2010). Many textbooks fail to enhance students' knowledge of the formal definition of limit, resulting in limit learning being mindlessly applied for calculation rather than conceptual understanding (Liang, 2016). Additionally, it is likely that some calculus teachers themselves do not fully comprehend the concepts of limit or have some misconceptions about it. This is revealed by Mastorides and Zachariades' (2004) study, which found that teachers' content knowledge of limit was insufficient, and that this had an impact on their pedagogical content knowledge, as the majority of them had difficulty understanding multi-quantified statements or failing to comprehend the modification of such statements caused by changes in the order of the quantifiers.

It was found that students are given little opportunities to develop their own knowledge of the subject matter, because lecturers do not pay enough attention to the necessary conditions for teaching the notion of limits. Students are taught limit information with little awareness of their own misconceptions about the idea of limit, and they are not given the opportunity to experience the disagreement between preconceptions and formal understandings of limit content. Most calculus classes, on one hand, place a greater emphasis on calculation rather than demonstrating conceptual knowledge, and as a result, students become passive receivers instead of critical thinkers. Based on the results of the independent sampling t-test in Table 3, the constructivist approach was found to have a significant impact on students' understanding of the concept of limits in a calculus course. The results show that the experimental group, who are taught using a constructive approach, performs better than the control group, who are taught using a conventional method. Students' performance is greatly improved when they learn using a constructivist method since they are actively engaged in the learning activities. In a constructivist approach, students' cognitive skills can also be boost because they are encouraged to develop their knowledge and explore beyond what the teacher has offered. This finding is consistent with those of Bokhari and Yushau (2006), Juter (2007), Klymchuk (2010), Liang (2016), Mastorides and Zachariades (2004), who argued that a new method to teaching limits will translate into better performance.

Gender was identified to have a significant impact on polytechnic students' post-test scores based on the findings of the independent sampling t-test statistic from Table 4. With a mean difference of 10.85 in their post-test scores, male students scored better than their female counterparts in the concept of limits. This is because students can understand and apply basic limits principles in a variety of scenarios after studying using a student's centred learning. The constructivist approach, according to Sani *et al.* (2014), allows students to actively participate in observations, analyzing trends, and forming conclusions based on the collected data. The findings of the study corroborated those of Beecher and Sweeny (2008), Carol (2005), Castle, Deniz, and Tortora (2005), Garba and Muhammad (2015).

Conclusion

In the more extensive review of student answers, some interesting trends emerged. Even though students had successfully answered to graphical problems about limit, their answers to mathematical notation and definition of tasks were low. This discrepancy could be explained by the students' prior knowledge with limit questions (they may have had more opportunities to consider and learn about limits in the presence of this representation than with the others). Due to the diversity of the population and the fact that many students had been taught by more than one instructor and textbook, there is a probability that many will be familiar with many representations of limits. It's possible that students can respond to limits questions without having a solid conceptual understanding of calculus or that student can demonstrate their solid understanding of the ideas when interpreting limits questions but can't do so when reading notation and graph type questions for some reason. As a result, more research is needed to determine what students who correctly answered the questions understand about limits and why this understanding isn't evident on the notation and graph type questions.

Recommendations

Instructional delivery of a topic has always been one of the master piece of action aimed at supporting students in performing effectively in the concept of limits. According to this study, students who were taught limits using a constructivist approach performed better academically than students who were taught using a conventional teaching method. The researchers propose the following recommendations based on the findings of this study:

(i) Conventional lecture method should be discouraged, whereas constructivist approach should be encouraged because they are directly students' centered.
(ii) Calculus lecturers should make the concept of limits attractive so that students can develop their own knowledge or at the very least interpret it based on their own experiences.
(iii) Proper strategies should be used in teaching the concept of limits in order to have a shift in pedagogical instruction because of its importance in the core conception of calculus course.

References

Bokhari, M. A., & Yushau, B. (2006). Local approximation of a function of single variable: an alternative way to define limit. *International Journal of Mathematical Education in Science & Technology*, 37(5), 515-526. https://doi.org/10.1080/00207390500503079

Baba, T. Y. (2021). Promoting technological and economic innovation in post-COVID 19: The role of the polytechnic education. Keynote speaker at the 8th ASUP annual national conference held at Waziri Umaru Federal Polytechnic Birnin Kebbi, from 16th – 18th November, 2021.

Beecher, M., & Sweeny, S. (2008). Closing the achievement gap with curriculum enrichment and differentiation: One school's story. *Journal of Advanced Academics*, 19(3), 502-530.

Carol, T. (2005). The effects of grouping practices and curricular adjustments on achievement. *Journal for the Education of the Gifted*, 29(1), 60-89.

Castle, S., Deniz, C. B., & Tortora, M. (2005). Flexible grouping and student learning in a high-needs school. *Education and Urban Society*, 37(39), 139-150.

Chukwuyenum, A. N. (2013). Impact of critical thinking on performance in mathematics among senior secondary school students in Lagos state. *Journal of Research and Methods in Education*, 3(5), 18-25.

Cohen, L., Manion, L., & Morrison, K. (2017). *Research methods in education* (8th Ed). Routledge, Taylor & Francis Group.

Garba, A. A., & Muhammad, S. A. (2015). The effectiveness of differentiated instruction on students' geometric achievement in Kebbi state senior secondary schools, Nigeria. *International Journal of Scientific and Engineering Research,* 6(1), 1905-1922. http://www.ijser.org

Juter, K. (2007). Students' concept development of limits. *CERME*; 5, 2320-2329.

Kamoru, K. (2021). Promoting technological and economic innovation in post-COVID 19: The role of polytechnic education. Lead paper presenter at the 8th ASUP annual national conference held at Waziri Umaru Federal Polytechnic Birnin Kebbi, from 16th – 18th November, 2021.

Klymchuk, S. (2010). *Counterexamples in calculus*. Washington, DC: MAA, 2010.

Liang, S. (2016). Teaching the concept of limit by using conceptual conflict strategy and Desmos graphing calculator. *International Journal of Research in Education and Science*, 2(1), 35-48. http://www.ijres.net

Mastorides, E., & Zachariades, T. (2004). Secondary mathematics teachers' knowledge concerning the concept of limit and continuity. Paper presented at the international group for the psychology of mathematics education, 28th, Bergen, Norway.

Micheal, K. O., & Iduma, A. I. (2013). Polytechnic education and human resources development in Nigeria: Challenges and constraints. *Journal of Educational and Social Research,* 5(1).

Muhammad, S. A., Abdullah, A. H., & Osman, S. (2020). Developing mathematical thinking among polytechnic students in linear algebra through peer tutoring strategy. *Journal of Advanced Research in Dynamical and Control Systems*, 12(3), 423-434. https://doi.org/10.5373/JARDCS/V12I3/20201210

Muhammad, S. A., Abdullah, A. H., & Osman, S. (2021). Improving polytechnic students' high-order-thinking-skills through inquiry-based learning in mathematics classroom. *International Journal of Evaluation and Research in Education*, 10(3), 976-983. https://doi.org/10.11591/ijere.v10i3.21771

Muhammad, S. A., Abdullah, A. H., Osman, S., Ashari, Z. M., Jumaat, N. F., Ali, D. F., & Abu-Samah, N. (2020). Polytechnic students' mathematical thinking processes in linear algebra: A qualitative approach. *Universal Journal of Educational Research*, 8(9), 3946-3957. https://doi.org/10.13189/ujer.2020.080919

Sani, M. A. (2017). *Introduction to research methodology and statistics*: *A guide for students and supervisors*. Ahmadu Bello University Press.

Sani, M. S., Garba, I. M., & Abdullahi, F. (2014). Effect of discovery approach on students' mathematics achievement in Kebbi state junior secondary schools. *Gesse Journal of Humanities,* 1(1), 161-166.

Stewart, J. (2012). *Calculus* (7th Ed.). Belmont, CA: Brooks/Cole.

Uwaifo, V. O. (2010). Technical education and its challenges in Nigeria in the 21st century. *International NGO Journal*, 5(2), 40-44.