

## **MATHEMATICAL MODELLING OF TRANSMISSION DYNAMICS OF ONCHOCERCIASIS**

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### **Abstract**

*In this paper, a mathematical model for the transmission dynamics and control of Onchocerciasis was developed incorporating the infectious but not blind and the infectious blind compartments. The model consisting of systems of coupled nonlinear ordinary differential equation are used to describe this spread. We obtain the effective reproductive number ( $R_c$ ), and its values computed using five (5) different control strategies were carried out and result shows that although, a 60% treatment coverage rate of infectious but not blind and infectious blind individuals only is better than 80% treatment coverage rate of infectious but not blind individuals only. A 40% coverage rate of fumigation and treatment of infectious but not blind is better than a 40% coverage rate of fumigation only. Further analysis revealed that a 30% coverage rate of fumigation and treatment of infectious blind is better than 80% coverage rate of fumigation only or fumigation and treatment of infectious but not blind only. Also, sensitivity analysis was carried out with respect to the model parameter values to determine the relative importance of each model parameter on the transmission and control of onchocerciasis. From the result, effective fumigation rate  $\rho$  has the highest sensitivity index followed by the effective contact rate  $\alpha_1$  while the negative sensitivity index is  $\gamma_2$ .*

**Keywords:** Blindness, Differential Transformation, Effective Reproductive number, Equilibrium state, Ivermectin, Onchocerciasis

### **Introduction**

Onchocerciasis (River blindness), infection is one of the neglected tropical diseases targeted for eradication by 2030. Onchocerciasis is a disease that causes visual deficiency (blindness) and it is the second largest cause of blindness after trachoma (Abah *et al.*, 2015). It primarily affects the eyes and the skin. It is transferred from one person to another through the *Onchocerca volvulus* parasite as a result of a bite from a blackfly that has ingested microfilaria of the class *Simulium damnosum sensulato*. The disease is usually transmitted to humans through the bites of blackflies (Hamley *et al.* 2020). These black flies mostly breed near well-oxygenated fast-running water bodies. The disease is common in mostly remote agricultural villages near rivers and streams. Onchocerciasis starts with Pruritus, which is often the first indication of *Onchocerca* infection. It may occur on its own or in association with onchocerca skin disease. Troublesome itching, of the body caused by movement of the tiny worms that moves under the skin, change in skin colour, development of white patches on the legs, (leopard skin) (Latipah *et al.* 2020). In the absence of any other pruritic skin condition, patients could be observed using wooden sticks to scratch their skin in an attempt to obtain relief (Abdon *et al.* 2015). Depigmentation (Leopard Skin) this is associated with the skins, calf of the leg, inguinal regions, external genitalia and auxiliary folds in older residents of endemic areas (Bako *et al.* 2024). The early age of depigmentation consists of small yellow brown macules resulting in a rather mottled appearance, others include fever and constant headache (Idowu and Ogunmiloro, 2020). Several mathematical models have been used to evaluate intervention strategies concerning onchocerciasis disease. Ogunmiloro and Idowu 2022 adopted the theory of optimal control and explored the effectiveness of controls such as personal protection, treatment with ivermectin and vector control used to combat

onchocerciasis diseases. Their results indicated that vector control was the best among the controls considered.

Also, researchers have conducted various studies exploring potential methods to mitigate the transmission of the disease. Melchers *et al.* 2020, utilized the skin snip survey in West Africa to assess the impact of mitigating black flies through larviciding. In a related study, Adeyemo *et al.* 2024, Asha and Nyimvua 2020, employed computer simulation models to investigate onchocerciasis prevention.

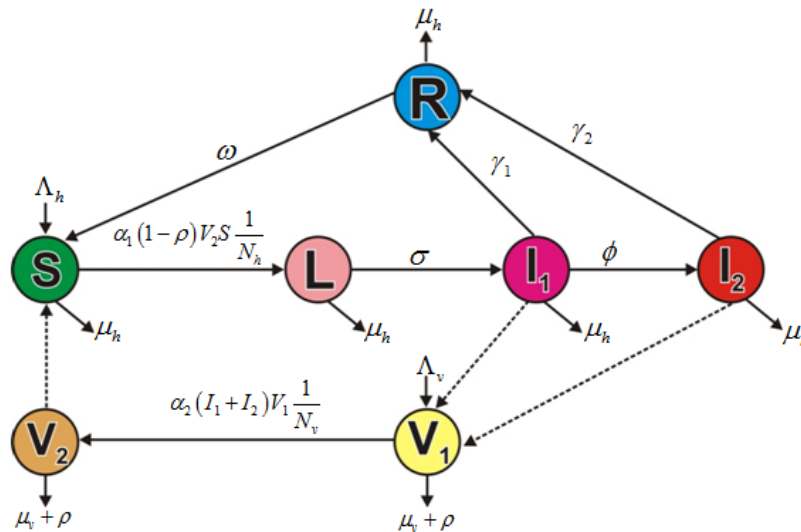
During the last decade, Hamley *et al.*, (2021), Melchers *et al.*, (2020), Latipah *et al.*, (2020), Ogunmiloro and Idowu (2022), Adeyemo *et al.*, (2024), Idowu and Ogunmiloro (2020) and Adeyemo (2023), have designed mathematical models on Onchocerciasis (river blindness). Considering the works of the afore mention authors, the study at hand is an improvement on the cited models above in that it includes;

- i. The latent, the infectious blind and recovered classes
- ii. The treatment of actively infected individuals and blind as control parameters
- iii. Incorporating the fumigation parameter
- iv. Loss of immunity after recovered.

## Material and Methods

### Model Variables and Parameters

The model describes the dynamics of two populations, which is compartmentalized into seven (7) classes interacting with each other namely: the human and vector population.



The model applied has the following variables and parameters:

- $S(t)$  Susceptible individuals at time  $t$
- $L(t)$  Latently infected individuals at time  $t$
- $I_1(t)$  Infectious but not blind individuals at time  $t$
- $I_2(t)$  Infectious blind individuals at time  $t$
- $R(t)$  Removed individuals due to recovery from infection at time  $t$
- $V_1(t)$  Non-carrier vectors at time  $t$

$V_2(t)$	Carrier vectors at time $t$
$\Lambda_h$	Recruitment rate of human through birth
$\alpha_1$	Effective contact Rate of Onchocerciasis
$\rho$	Effective Fumigation Rate
$\sigma$	Rate of Infection from $L$ to $I_1$ individuals
$\gamma_1$	Treatment rate of $I_1$ individuals
$\gamma_2$	Treatment rate of $I_2$ individuals
$\mu_h$	Mortality rate due to Onchocerciasis
$\omega$	Waning Rate due of Temporal Immunity of Recovered individuals
$\mu$	Natural death rate of humans
$\alpha_2$	Progression Rate of infection in the vector
$\mu_v$	Natural death rate of vectors
$\Lambda_v$	Recruitment rate of blackfly through birth

### Model Development

The Susceptible human population ( $S$ ) is generated at a constant rate  $\Lambda_h$  via recruitment of humans by birth and immigration into the population. It is further increased by recovered individuals losing immunity at the rate  $\omega$ . It is decreased by infection acquired via contact with carrier vectors at a rate  $\alpha_1$  where  $\rho$  (i.e  $0 < \rho < 1$ ) is the effective fumigation rate. It is further decreased by natural death at the rate  $\mu_h$ . All the newly infected susceptible individuals moved to the latently infected compartment ( $L$ ). The Latent compartment ( $L$ ) is decreased by progression of the disease to infectious but not blind class  $I_1$  and natural death at the rates  $\sigma$  and  $\mu_h$  respectively. It is further decrease due to following interaction with the non-carrier vector ( $V_1$ ). The infectious but not blind ( $I_1$ ) population is increased by progression of latently infected individuals at the rate  $\sigma$  and decreases by complications of the infection resulting into blindness at the rate  $\phi$ , it further decreases by recovery due to treatment to the recovered class at the rate  $\gamma_1$  and natural death also occurs at the rate  $\mu_h$ . The infectious blind compartment ( $I_2$ ) is increased at the rate  $\phi$  due to progression of infection from  $I_1$  class and diminished by recovery due to treatment from *Onchocerca volvulus* (not the blindness) and natural death at the rate  $\gamma_2$  and  $\mu_h$  respectively. The recovered compartment ( $R$ ) is generated from recovered individuals from the  $I_1$  and  $I_2$  classes at the rates  $\gamma_1$  and  $\gamma_2$  respectively. It diminishes by losing immunity and natural death at the rates  $\omega$  and  $\mu_h$  respectively. The non-carrier vectors, ( $V_1$ ) are generated via birth or immigration of vectors into the population at the rate  $\Lambda_v$ . It decreases by infection acquired when the non-carrier vectors feed from the blood of the  $I_1$  or the  $I_2$  individuals at the rate  $\alpha_2$ . The vectors sub-population diminishes by natural death or larvicides at the rates  $\mu_v$  and  $\rho$  respectively. The Carrier vectors ( $V_2$ ) are generated from the Non-carrier vectors ( $V_1$ ) that feed on infectious blood meals and similarly diminishes at the rates  $\mu_v$  and  $\rho$  respectively.

Mathematical representation of the Schematic diagram is given by the following ordinary differential equations.

The corresponding mathematical model equations are described by a system of Ordinary Differential Equations (ODEs) given below:

$$\left. \begin{aligned} \frac{dS}{dt} &= \Lambda_h - \frac{\alpha_1(1-\rho)V_2S}{N_h} + \omega R - \mu_h S \\ \frac{dL}{dt} &= \Lambda_h - \frac{\alpha_1(1-\rho)V_2S}{N_h} - (\sigma + \mu_h)L \\ \frac{dI_1}{dt} &= \sigma L - (\phi + \gamma_1 + \mu_h)I_1 \\ \frac{dI_2}{dt} &= \phi I_1 - (\gamma_2 + \mu_h)I_2 \\ \frac{dR}{dt} &= \gamma_1 I_1 + \gamma_2 I_2 - (\omega + \mu_h)R \\ \frac{dV_1}{dt} &= \Lambda_v - \frac{\alpha_2(I_1 + I_2)V_1}{N_v} - (\mu_h + \rho)V_1 \\ \frac{dV_2}{dt} &= \frac{\alpha_2(I_1 + I_2)V_1}{N_v} - (\mu_h + \rho)V_2 \end{aligned} \right\}$$

(1)

where

$$N_h(t) = S(t) + L(t) + I_1(t) + I_2(t) + R(t) \tag{2}$$

$$N_v(t) = V_1(t) + V_2(t) \tag{3}$$

So that

$$\frac{dN_h(t)}{dt} = \Lambda_h - \mu_h N_h(t) \tag{4}$$

$$\frac{dN_v(t)}{dt} = \Lambda_v - (\mu_v + \rho) N_v(t) \tag{5}$$

in the biological feasible region:

$$\Omega = \left( \begin{array}{c} S \\ L \\ I_1 \\ I_2 \\ R \\ V_1 \\ V_2 \end{array} \right) \in \mathbb{R}_+^7 \left\{ \begin{array}{l} S \geq 0, \\ L \geq 0, \\ I_1 \geq 0, \\ I_2 \geq 0, \\ R \geq 0, \\ V_1 \geq 0, \\ V_2 \geq 0, \\ S + L + I_1 + I_2 + R + V_1 + V_2 \end{array} \right. \tag{6}$$

Setting

$$\left. \begin{aligned} \theta &= 1 - \rho \\ k_1 &= \sigma + \mu_h \\ k_2 &= \phi + \gamma_1 + \mu_h \\ k_3 &= \gamma_2 + \mu_h \\ k_4 &= \omega + \mu_h \\ k_5 &= \mu_v + \rho \end{aligned} \right\} \quad (7)$$

System (1) becomes

$$\left. \begin{aligned} \frac{dS}{dt} &= \Lambda_h - \frac{\alpha_1 \theta V_2 S}{N_h} + \omega R - \mu_h S \\ \frac{dL}{dt} &= \frac{\alpha_1 \theta V_2 S}{N_h} - k_1 L \\ \frac{dI_1}{dt} &= \sigma L - k_2 I_1 \\ \frac{dI_2}{dt} &= \phi I_1 - k_3 I_2 \\ \frac{dR}{dt} &= \gamma_1 I_1 + \gamma_2 I_2 - k_4 R \\ \frac{dV_1}{dt} &= \Lambda_v - \frac{\alpha_2 (I_1 + I_2) V_1}{N_v} - k_5 V_1 \\ \frac{dV_2}{dt} &= \frac{\alpha_2 (I_1 + I_2) V_1}{N_v} - k_5 V_2 \end{aligned} \right\} \quad (8)$$

It can be seen that all solutions of the system starting in  $\Omega$  remains in  $\Omega$  for all  $t \geq 0$ , thus is positively invariant and sufficient to the solutions in  $\Omega$ .

### Existence of Onchocerciasis-free Equilibrium State ( $E^0$ )

At the disease-free equilibrium state there is absence of infection, there is no infection

$$\begin{pmatrix} S \\ L \\ I_1 \\ I_2 \\ R \\ V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} S^0 \\ L^0 \\ I_1^0 \\ I_2^0 \\ R^0 \\ V_1^0 \\ V_2^0 \end{pmatrix} \quad (9)$$

Now, substituting and solving simultaneously yields

$$V_1^0 = \frac{\Lambda_v}{k_5} \quad (10)$$

$$S^0 = \frac{\Lambda_h}{\mu_h} \tag{11}$$

Thus a onchocerciasis-free equilibrium state of the model exists at the point

$$\begin{pmatrix} S \\ L \\ I_1 \\ I_2 \\ R \\ V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} \frac{\Lambda_h}{\mu_h} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{\Lambda_v}{k_5} \\ 0 \end{pmatrix} \tag{12}$$

**Effective Reproductive number ( $R_c$ )**

One of the most important concern in the analysis of epidemiological models is the determination of the asymptotic behaviour of their solution which is usually based on the stability of the associated equilibria. These models typically consist of disease-free equilibrium and at least one endemic equilibrium. The local stability is determined based on a threshold parameter known as basic reproduction number  $R_0$  this represents the average number of secondary cases generated by an infected individuals if introduced into a susceptible population with no immunity to the disease in the absence of interventions to control the infection. If  $R_0 < 1$ , then on average, an infected individual produces less than one newly infected individual over the course of its infection period. In this case, the infection may die out in the long run. Conversely, if  $R_0 > 1$ , each infected individual produces, on average more than one new infection, the infection will be able to spread in a population. A large value of ( $R_0$ ) may indicate the possibility of a major epidemic. Similarly, the effective reproductive number ( $R_c$ ) represents the average number of secondary cases generated by an infected individual if introduced into a susceptible population where control strategies are used (Bako *et al.*, 2024).

A better widely accepted and used method for finding ( $R_0$ ) that reflect its biological meaning is the next generation operator approach described by Diekmann and Heesterbeek (2000) and subsequently analysed by Van de Driessche and Watmough (2002), Heesterbeek (1996). Using this technique we obtained the effective reproductive number, ( $R_c$ ) of the system (1) which is the spectral radius ( $\rho$ ) of the next generation matrix, ( $K$ ), that is ( $R_c$ )  $R_c = \rho(FV^{-1})$ . Both  $F$  and  $V$  are obtained from the Jacobian matrix of the linearization of the system (1) about the disease-free equilibrium.  $F$  is the matrix of the new infection terms and  $V$  the matrix of the transition terms. Noting that, the matrices  $F$  and  $V$  are formed from the coefficients of the infected classes, i.e. ( $L, I_1, I_2, V_2$ ), in the system(1).

Then,

$$F = \begin{pmatrix} 0 & 0 & 0 & \alpha_1\theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & \alpha_2 & 0 \end{pmatrix} \quad (13)$$

and

$$V = \begin{pmatrix} k_1 & 0 & 0 & 0 \\ -\sigma & k_2 & 0 & 0 \\ 0 & -\phi & k_3 & 0 \\ 0 & 0 & 0 & k_5 \end{pmatrix} \quad (14)$$

$$R_c = \sqrt{\frac{\alpha_1\alpha_2\theta\sigma(k_3 + \phi)}{k_1k_2k_3k_5}} \quad (15)$$

**Solution via Differential Transformation Methods (DTM)**

Differential Transform Methods is used to solve the model equation of the system (1) passing through the initial value conditions of the variables.

Let

$$S = y_1, L = y_2, I_1 = y_3, I_2 = y_4, R = y_5, V_1 = y_6, V_2 = y_7 \quad (16)$$

Substituting this into system (1) with initial condition

With initial conditions;

$$y_1(0) = 175,914,033, y_2(0) = 2,500,000, y_3(0) = 1,997,827, \quad (17)$$

$$y_4(0) = 300,000, y_5(0) = 850,196, y_6(0) = 2,776,642,857, y_7 = 500,000$$

Where

$\Lambda_h, \alpha_1, \alpha_2, \rho, N_h, \omega, \mu_h, \sigma, \phi, \gamma_1, \gamma_2, \Lambda_v, \mu_h, N_v$  are real.

Substituting the values of parameters into our model equations we have; yields

$$\frac{dy_1}{dt} = 3449679 - 0.000000002335y_7y_1 + 0.019y_5 - 0.019y_1 \quad (18)$$

$$\frac{dy_2}{dt} = \Lambda_h - \frac{\alpha_1(1-\rho)y_7y_1}{N_h} + (\delta + \mu_h)y_2$$

$$\frac{dy_2}{dt} = 0.000000002335y_7y_1 - 0.519y_2 \quad (19)$$

$$\frac{dy_3}{dt} = \delta y_2 - (\phi + \gamma_1 + \mu_h)y_3$$

$$\frac{dy_3}{dt} = 0.5y_2 - 1.586y_3 \quad (20)$$

$$\frac{dy_4}{dt} = \phi y_3 - (\gamma_2 + \mu_h)y_4$$

$$\frac{dy_4}{dt} = 1.5y_3 - 0.086y_4 \quad (21)$$

$$\frac{dy_5}{dt} = y_1y_3 + y_2y_4 - (\omega + \mu_h)y_5$$

$$\frac{dy_5}{dt} = 0.067y_3 + 0.067y_4 - 0.038y_5 \quad (22)$$

$$\frac{dy_6}{dt} = \Lambda_v - \frac{\alpha_2 y_3 y_6}{N_v} + \frac{\alpha_2 y_4 y_6}{N_v} - (\mu_v + \rho)y_6$$

$$\frac{dy_6}{dt} = 361,029 - 0.0000000000144y_3y_6 - 0.0000000000144y_4y_6 - 0.80013y_6 \quad (23)$$

$$\frac{dy_7}{dt} = \frac{\alpha_2 y_3 y_6}{N_v} + \frac{\alpha_2 y_4 y_6}{N_v} - (\mu_v + \rho)y_7$$

$$\frac{dy_7}{dt} = 0.0000000000144y_3y_6 + 0.0000000000144y_4y_6 - 0.80013y_6 \quad (24)$$

We take the differential transform of the above equations and obtained;

$$(k+1)Y_1(k+1) = 34496798\delta(k) - 0.000000002335 \sum_{r=0}^k Y_1(r)Y_7(k-r) + 0.019Y_5(k) - 0.019Y_1(k) \quad (25)$$

$$(k+1)Y_2(k+1) = 0.000000002335 \sum_{r=0}^k Y_1(r)Y_7(k-r) - 0.519Y_2(k) \quad (26)$$

$$(k+1)Y_3(k+1) = 0.5Y_2(k) - 1.586Y_3(k) \quad (27)$$

$$(k+1)Y_4(k+1) = 1.5Y_3(k) - 0.086Y_4(k) \quad (28)$$

$$(k+1)Y_5(k+1) = 0.067Y_3(k) - 0.067Y_4(k) - 0.038Y_5(k) \quad (29)$$

$$(k+1)Y_6(k+1) = 3610298(k) - 0.0000000000144 \sum_{r=0}^k Y_3(r)Y_7(k-r) - \left. \begin{aligned} &0.0000000000144 \sum_{r=0}^k Y_3(r)Y_7(k-r) - 0.80013Y_6(k) \end{aligned} \right\} \quad (30)$$

$$(k+1)Y_7(k+1) = 0.0000000000144 \sum_{r=0}^k Y_3(r)Y_6(k-r) + \left. \begin{aligned} &0.0000000000144 \sum_{r=0}^k Y_4(r)Y_6(k-r) - 0.80013Y_7(k) \end{aligned} \right\} \quad (31)$$

$$\text{For } k = 0, 1, 2, 3, \dots \quad (32)$$

When  $k = 0$ , we have the first iteration as follows;

$$Y_1(1) = -819,913.5365 \quad (33)$$

$$Y_2(1) = -1,092,120.366 \quad (34)$$

$$Y_3(1) = -1,918,553.622 \quad (35)$$

$$Y_4 = 2,970,940.5 \quad (36)$$

$$Y_5(1) = 81,446.961$$

$$Y_6(1) = -2,221,767.125 \quad (37)$$

$$Y_7(1) = -308,189.4731 \quad (38)$$

When  $k = 1$ , we have the second iteration as follows;

$$Y_1(2) = 72,337.3903$$

$$Y_2(2) = -127,548.9298 \tag{39}$$

$$Y_3(2) = 1,794,443.114$$

$$Y_4(2) = 1,566,665.658 \tag{40}$$

$$Y_5(2) = -165,345.5453 \tag{41}$$

$$Y_6(2) = 102,293.4412$$

$$Y_7(2) = 84,908.4678 \tag{42}$$

When  $k = 2$ , we have the third iteration as follows;

$$Y_1(3) = -11,261.4224 \tag{43}$$

$$Y_2(3) = 24,252.6539 \tag{44}$$

$$Y_3(3) = -938,033.1821 \tag{45}$$

$$Y_4(3) = 942,132.6392 \tag{46}$$

$$Y_5(3) = 77,159.1395 \tag{47}$$

$$Y_6(3) = -30,308.3807 \tag{48}$$

$$Y_7(3) = -19,620.2404 \tag{49}$$

and so on;

A closed form of this solution can be written as follows;

$$y(t) = \sum_{k=0}^{\infty} Y(k)t^k \tag{50}$$

The solution for  $y_1(t)$  is given below;

$$y_1(t) = Y_1(0) + tY_1(1) + t^2Y_1(2) + t^3Y_1(3) + \dots \tag{51}$$

$$y_1(t) = 175,914,033 - 819,913.5365t + 72,337.3903t^2 - 11,261.4224t^3 + \dots \tag{52}$$

The solution for  $y_2(t)$  is given below;

$$y_2(t) = Y_2(0) + tY_2(1) + t^2Y_2(2) + t^3Y_2(3) + \dots$$

$$y_2(t) = 250,000 - 1,092,120.366t + 63,774.4649t^2 + 24,252.6539t^3 + \dots \tag{53}$$

The solution for  $y_3(t)$  is given below;

$$y_3(t) = Y_3(0) + tY_3(1) + t^2Y_3(2) + t^3Y_3(3) + \dots \tag{54}$$

$$y_3(t) = 1,997,827 - 1,918,553.622t + 1,794,443.114t^2 - 938,033.1821t^3 + \dots \tag{55}$$

The solution for  $y_4(t)$  is given as follow

$$y_4(t) = 300,000 + 2,970,940.5t + 1,566,665.658t^2 + 942,132.6392t^3 + \dots \tag{56}$$

The solution for  $y_5(t)$  is given below;

$$y_5(t) = Y_5(0) + tY_5(1) + t^2Y_5(2) + t^3Y_5(3) + \dots \tag{57}$$

$$y_5(t) = 850,196 + 81,446.961t + 165,345.5453t^2 + 77,159.1395t^3 + \dots \quad (58)$$

The solution for  $y_6(t)$  is given below;

$$y_6(t) = Y_6(0) + tY_6(1) + t^2Y_6(2) + t^3Y_6(3) + \dots \quad (59)$$

$$y_6(t) = 2,776,642,857 - 2,221,767.125t + 102,293.4412t^2 - 30,308.3807t^3 + \dots \quad (60)$$

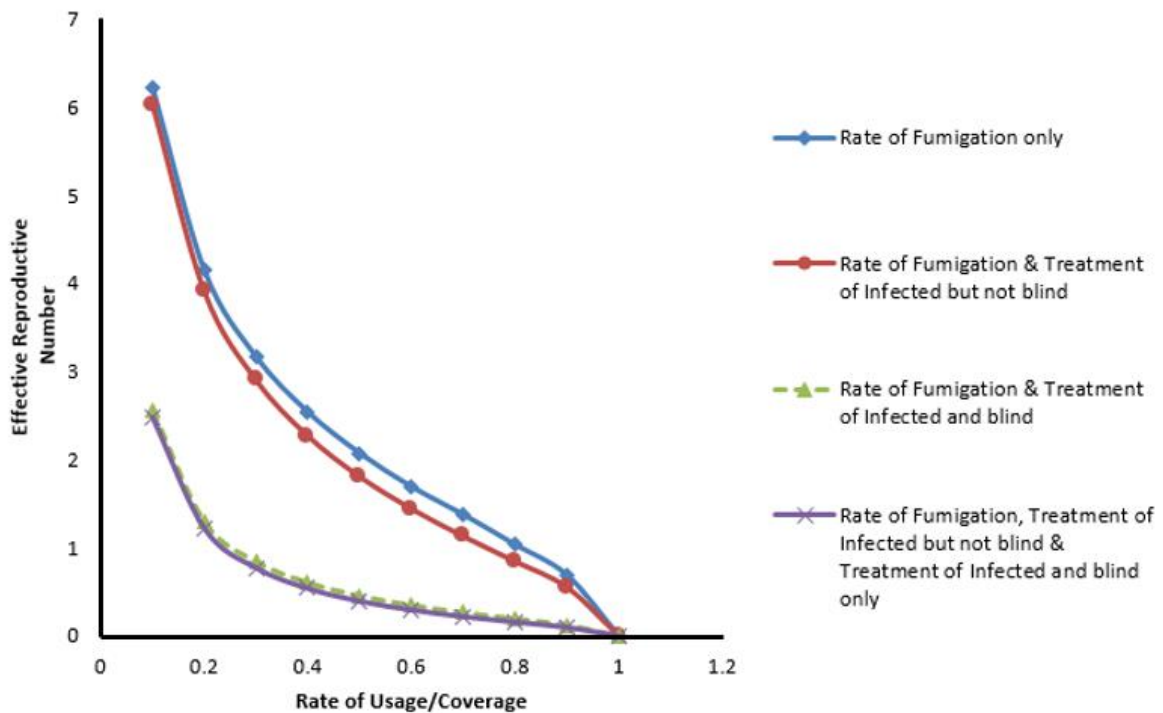
The solution for  $y_7(t)$  is given below;

$$y_7(t) = Y_7(0) + tY_7(1) + t^2Y_7(2) + t^3Y_7(3) + \dots \quad (61)$$

$$y_7(0) = 500,000 - 308,189.4731t + 84,908.4678t^2 - 19,620.2404t^3 + \dots \quad (62)$$

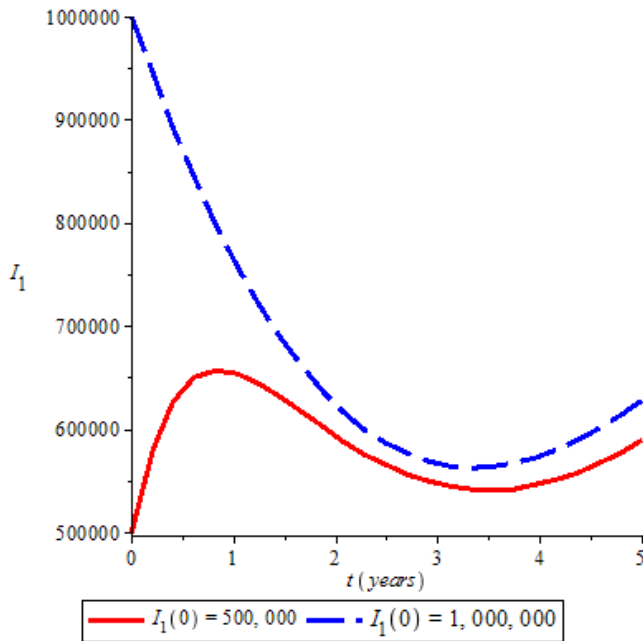
## Results and Discussion

In this section, we presented some numerical simulation to monitor the dynamics of the full model (1) for various values of effective reproductive number in order to confirm our analytical results on the global stability of the Onchocerciasis-free equilibrium as well as effect of different control strategies.



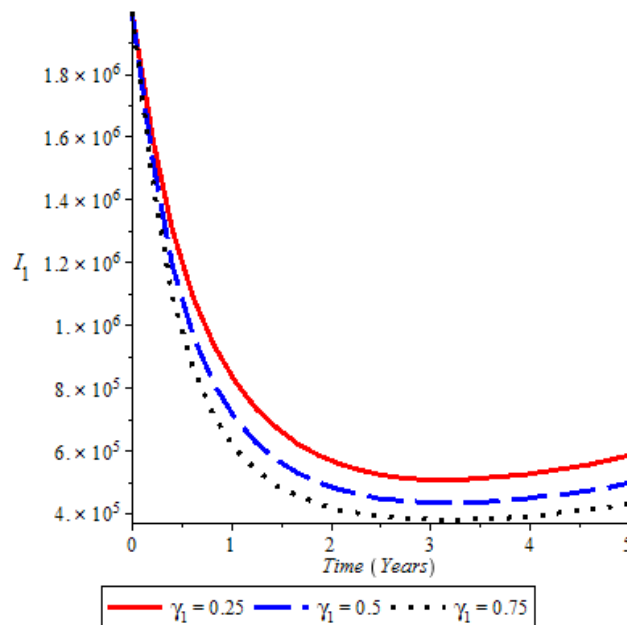
**Figure 2: Comparison between the four control strategies and the Effective Reproduction Number at Different Coverage Rate**

Figure 2 reveals that the four control strategies reduce the effective reproductive number below unity. Although, a 40% coverage rate of fumigation and treatment of Infectious but not blind is better than a 40% coverage rate of fumigation only. It further reveals that a 30% coverage rate of fumigation and treatment of Infectious but not blind is better than 80% coverage rate of fumigation only or fumigation and treatment of Infectious but not blind only.



**Figure 3: Total number of infectious but not blind with different initial variable conditions**

In Figure 3, the  $I_1 = 50000$  and  $I_1 = 1,000,000$ . Control Parameters used are  $I_1 = 500,000$ , and  $I_1 = 1,000,000$  which gives  $R_C = 14.96388$  when  $\gamma_1 = 0.0001$ ,  $\gamma_2 = 0.0001$ ,  $\rho = 0.0001$ . It is clear from the above that the solutions profiles of equation 1 converges to the endemic equilibrium in each of these two different initial values of  $I_1(0)$ . This shows the local and the global asymptotic stability of the endemic equilibrium.



**Figure 4: A comparison between the effects of treatment of Infectious but not blind on the morbidity of  $I_1$  individuals.**

In Figure 4, the Initial parameters and values used are  $\gamma_1 = 0.25$ ,  $\gamma_1 = 0.5$  and  $\gamma_1 = 0.75$ . This shows that more of the Infectious individuals should be treated with Ivermectin annually, since 75% treatment rate of Infectious but not blind reduces the morbidity to a disease-free equilibrium in the first three (3) years. Hence, it is advice that 75% treatment will reduce onchocerciasis to a disease-free equilibrium state.

### **Conclusion**

Here we present a model mathematically for curbing of Onchocerciasis by integrating the infectious but not blind and the infectious blind classes. It is proved that the model is mathematically well posed and epidemiologically meaningful in this feasible region. The effective reproductive number  $R_c$ , which is a threshold for disease control is obtained. A sensitivity analysis of the  $R_c$  using the model parameters was carried out and found that the highest sensitivity parameters of the Onchocerciasis model are identified, effective fumigation rate  $\rho$  denotes the most sensitivity index next followed by the effective contact rate  $\alpha_1$  and the smallest sensitive is  $\mu_v$ .

Also, the Differential Transformation Method (DTM) has been successfully applied to approximately solve a system of nonlinear equations on Onchocerciasis dynamics and control. The result shows the potential efficiency of DTM in solving nonlinear problems. It can thus be concluded that when combined with high performance computer and symbolic computation software such as Maple, Matlab, Mathematica and so on, the Differential Transformation Method stand a chance of becoming a new powerful analytic tool to obtain satisfactory non-linearized and unperturbed approximations for nonlinear problems unrestrictive assumptions in science and engineering.

### **References**

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