MULTIVARIATE TIME SERIES ANALYSIS ON THE PRICES OF STAPLE FOODSTUFFS IN KWARA STATE, NIGERIA

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Abstract
Due to supplementary, complementary and substitute relationship between staple foodstuffs, the prices of one or more staple foodstuffs tend to influence and could be used to predict the prices of some others. This study was therefore aimed at establishing the co-movement between the prices of some major stable foodstuffs - Rice, Maize, Garri, Millet, Guinea-Corn and Beans - in Kwara State, Nigeria. Multivariate time series models were fitted to data on monthly prices of Rice, Maize, Garri, Millet, Guinea-Corn and Beans over a period of twelve years (from January 2000 to December 2012). The cointegration relations among the prices were established by applying Johansen's cointegration tests. As a result, appropriate Vector Error Correction (VEC) model was fitted to the data. The unit root test for stationarity in the series reveals that all the series were non-stationary but they were only made to be stationary at first difference. The results from the analysis showed that there exist short term adjustments and long-term dynamics among the prices of Rice, Maize, Garri, Millet, Guinea-Corn and Beans in Nigeria over the study period. Further results showed that a Vector Error Correction (VEC) model of lag two with one cointegration equation best fits the data. The forecasting accuracy of the fitted model was determined by out-of-sample forecasts of the future prices of the selected staple foodstuffs. Suitable model's assessment criteria such as root mean square error, mean absolute error and the like were employed to determine the efficiency of the fitted model. The data employed for the study were collected from the Kwara State office of National Bureau of Statistics, Nigeria. All analyses were performed in the environment of R Statistical package.

Keywords: *Prices, Vector autoregressive, co-integration, Vector Error correction model and forecasting.*

Introduction

Important food grain in Nigeria whose production is being emphasized to remedy food deficit and importation includes maize, guinea corn, millet, rice and pulses (Aihonsu and Akorede, 2002). Prices are a standard and important component of market and food security analysis because they serve as an indicator of both food availability and food access. Prices are a measure of availability because they tend to rise as the supply of food falls in relation to demand (e.g., poor production, constrained imports of food), and they tend to fall when supply expands in relation to demand (e.g., a bumper harvest). Food prices are also a measure of food access because they affect the household's purchasing power: the ability of a household to acquire goods and services based on the amount of money or other forms of wealth they possess. Consumer prices of food determine how much food a household can buy given their level of income or wealth (FEWS NET, 2009).

Rice, Maize, Garri, Millet, Guinea-Corn and Beans are among staple food items whose prices are highly unstable between seasons in Kwara State.

This work examines the relationship between price levels of staple foodstuffs in Kwara state and seeks to determine whether or not they are linked. This study therefore analyzes the trend in price of staple foodstuffs and integration between prices of these staple food items in Kwara state and determines the causal relationship between and among the series. The study is based on the assumptions that there is no causal relationship between prices of staple foodstuffs.

When large records of price quotations are available and reasonably accurate, a great deal can be learnt from them about the relationship and future prices of staple food stuff over time and space. Research into the workings of major commodity future prices, relying largely on price of the commodities in the previous years, has amply demonstrated the value of approaching the study of the relationships and future prices through price analysis. Reported here is an attempt to employ, in a similar fashion, much less plentiful and less reliable information about the retail prices of staple foodstuffs in Nigeria to reveal salient features regarding the relationships and future prices of staple food.

The accurate measurement of the stochastic component in the prices may contribute to policy decisions regarding the possible implementation of commodity price stabilization programmes. The focus of the present work is to fit appropriate time series model on the prices of major staple foodstuffs in Nigeria. The selected foodstuffs for this study are Rice, Maize, Garri, Millet, Guinea-Corn and Beans. Results from this work shall be useful to establish the complementary relationships between the prices of these foodstuffs. By this, the future prices of these commodities can be efficiently predicted for policy formulation by interested stakeholders..

Materials and Methods

Data Description

The data for this study were obtained from National Bureau of Statistics (NBS), Kwara State office, Nigeria. The data were on monthly retail prices of staple foodstuffs (maize, garri, millet, guinea corn, palm oil and beans) in Nigeria over a period of twelve years covering between January 2000 and December 2012 inclusive.

Vector Autoregression (VAR) Model

The following structure was defined to develop a suitable time series model for the data on the prices of selected foodstuffs in Nigeria as collected.

Let $Y_t = (y_{1t}, y_{2t}, \ldots, y_{nt})^T$ denote a ($n \times 1$) vector of time series variables. The basic p-lag vector autoregressive (VAR (p)) model has the form (Hamilton,1994; Gujarati, 1979; 2004)

$$Y_t = c + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \ldots + \Pi_p Y_{t-p} + \varepsilon_t ; \quad t = 1, 2, \ldots, T \qquad (1)$$

where c denotes a n x 1 vector of constants and Π_j a n x n matrix of autoregressive coefficients for j = 1, 2, . . . , p. The n x 1 vector ε_t, is a vector generalization of white noise: $\Pi_{11}^{(1)} Y_{1,t-1}$

$$E(\varepsilon_t) = 0 \text{ and } E(\varepsilon_t \varepsilon_s') = \begin{cases} \Sigma, & \text{if } t = s \\ 0, & \text{if } t \neq s \end{cases} \qquad (2)$$

with Σ an (n x n) symmetric positive definite matrix.
Let c_i denote the i^{th} element of the vector c and let $\Pi_{ij}^{(1)}$ denote the row i, column j element of the matrix Π_1.Then, the first row of the vector system in (1) specifies that

$$Y_{1t} = c + \Pi_{11}^{(1)} Y_{1,t-1} + \Pi_{12}^{(1)} Y_{2,t-1} + \ldots + \Pi_{1n}^{(1)} Y_{n,t-1} + \Pi_{11}^{(2)} Y_{1,t-2} + \Pi_{12}^{(2)} Y_{2,t-2} + \ldots + \Pi_{1n}^{(2)} Y_{n,t-2} + \Pi_{11}^{(p)} Y_{1,t-p} + \Pi_{12}^{(p)} Y_{2,t-p} + \Pi_{1n}^{(p)} Y_{n,t-p} + \varepsilon_{1t}; t = 1, 2 \ldots, T \qquad (3)$$

Thus, a vector auto-regression is a system in which each variable is regressed on a constant and p of its own lags as well as on p lags of each of the other variables in the VAR. Note that each regression has the same explanatory variables.
Using lag operator notation, equation (1) can be written in the form

$$\Pi(L) Y_t = c + \varepsilon t \qquad (4)$$

Where $\Pi(L) = \mathbf{I}_n - \Pi_1 L - \ldots - \Pi_p L^p$ the VAR is stable if the roots of

$$\text{Det.}(\mathbf{I}_n - \Pi_1 Z - \ldots - \Pi_p Z^p) = 0 \qquad (5)$$

lie outside the complex unit circle (have modulus greater than one), or, equivalently, if the eigen values of the companion matrix

$$F = \begin{pmatrix} \Pi_1 & \Pi_2 & \cdots & \Pi_n \\ I_n & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & I_n & 0 \end{pmatrix} \quad (6)$$

have modulus less than one. Assuming that the process has been initialized in the infinite past, then a stable VAR (p) process is stationary with time invariant means, variances, and autocovariances.

If Y_t in (1) is covariance stationary, then the unconditional mean is given by

$$\mu = (\mathbf{I}_n - \Pi_1 - \cdots - \Pi_p)^{-1}c$$

The mean-adjusted form of the VAR (p) is then

$$Y_t - \mu = \Pi_1(Y_{t-1} - \mu) + \Pi_2(Y_{t-2} - \mu) + .. + \Pi_p(Y_{t-p} - \mu) + \varepsilon_t \quad t = 1, 2 \ldots. T \quad (7)$$

The basic VAR (p) model may be too restrictive to represent sufficiently the main characteristics of the data. In particular, other deterministic terms such as a linear time trend or seasonal dummy variables may be required to represent the data properly. Additionally, exogenous variables may be required as well. The general form of the VAR (p) model with deterministic terms and exogenous variables is given by

$$Y_t = \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \cdots + \Pi_p Y_{t-p} + \Phi D_t + G X_t + \varepsilon_t;\ t = 1, 2, \ldots, T \quad (8)$$

where D_t represents an $(l \times 1)$ matrix of deterministic components, X_t represents an $n \times 1$ vector of exogenous variables, and Φ and G are parameter matrices.

Testing Stationarity: Unit root test

Before fitting a particular model to time series data, the series must be made stationary. Stationarity occurs in a time series when the mean and autocovariances of the series remains constant over the time series. Therefore, the stochastic process Y_t is said to be stationary if:

(i) $E(Y_t) = \boldsymbol{\mu}$, constant for all value of t (9)

(ii) The $Cov(Y_t, Y_{t-j}) = \Gamma j = E[(Y_t - \mu)(Y_{t-j} - \mu)^T] = \Gamma\text{-}j^T$ for all t and j=0,1,2… (10)

Condition (9) means that all Yt have the same finite mean vector μ and [3.10] requires that the autocovariances of the process do not depend on t but just on the time period j the two vectors Yt and Y_{t-j} are apart. Therefore, a process is stationary if its first and second moments are time invariant.

Frequently, differencing may be needed to achieve stationarity. To test for stationarity of a series several procedures has been developed. The most popular ones are Augmented Dickey- Fuller (ADF) test due to Dickey and Fuller (1979, 1981). The following discussion outlines the basics features of unit root tests (Hamilton, 1994).

Consider a simple AR (1) process:

$$Y_t = \rho Y_{t-1} + X_t \delta + \varepsilon_t \quad (11)$$

where X_t are optional exogenous regressors which may consist of constant or a constant and trend, ρ and δ are parameters to be estimated, and ε_t assumed to be white noise. If $|\rho| = 1$, Y_t is a non-stationary series and the variance of y increases with time and approaches infinity. If $|\rho|<1$, Y_t is a stationary series. Thus, the hypothesis of (trend) stationarity can be evaluated by testing whether the absolute value of ρ is strictly less than one.

Hypothesis:

The series are not stationary ($\rho = 1$) against H_1: The series are stationary ($\rho < 1$)

Testing for cointegration using Johansen's methodology

The starting point in Johansen's procedure (Johansen, 1988; 1991), in determining the number of cointegrating vectors, is the VAR representation of Y_t. It is assumed a vector autoregressive model of order p and is expressed as follows:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_p Y_{t-p} + B X_t' \delta + \varepsilon_t$$

Where y_t is a *k*-vector of non-stationary I(1) variables(If a non-stationary series, y_t must be differenced d times before it becomes stationary, then it is said to be integrated of order d. This would be written $y_t \sim I(d)$), X_t is a d-vector of deterministic variables, and ε_t is a vector of innovations.

We rewrite this VAR as:

$$\Delta y_t = \Pi Y_{t-1} + \sum_{i-1}^{p-1} \Gamma_i \Delta y_{t-i} + BX_t'\delta + \varepsilon_t \qquad (12)$$

where

$$\Pi = \sum_{i-1}^{p-1} A_i - I, \quad \Gamma_i = -\sum_{j=i+1}^{p} A_j \qquad (13)$$

Granger's representation theorem asserts that if the coefficient matrix Π has reduced rank $r<k$, then there exist $k \times r$ matrices α and β each with rank r such that $\Pi=\alpha\beta'$ and $\beta' y_t$ is I(0), where r is the number of cointegrating relations (the *cointegrating rank*) and each column of Π is the cointegrating vector. The elements of α are known as the adjustment parameters in the VEC model. It can be shown that for a given r, the maximum likelihood estimator of β defines the combination of $Yt-1$ that yields the r largest canonical correlations of ΔYt with $Yt-1$ after correcting for lagged differences and deterministic variables when present.

Johansen (1988) proposed two tests for estimating the number of cointegrating vectors: the Trace statistics and Maximum Eigenvalue. Trace statistics investigate the null hypothesis of r cointegrating relations against the alternative of n cointegrating relations, where n is the number of variables in the system for r = 0, 1, 2…n-1. Define λ_i, i=1,2,…,k to be a complex modulus of eigenvalues of $\hat{\Pi}$ and let them be ordered such that $\lambda_1 > \lambda_2 > \ldots > \lambda_n$. The trace statistic computed as:

$$\lambda_{trace}(r) = -T\sum_{i=r+1}^{n} \log[1-\lambda_i] \qquad (14)$$

The Maximum Eigenvalue statistic tests the null hypothesis of r cointegrating relations against the alternative of r+1 cointegrating relations for r = 0, 1, 2 ,…, n-1. This test statistic is computed as (Engle et al., 1987):

$$\lambda_{max}(r, r+1) = -T\sum_{i=r+1}^{n} \log(1-\lambda_{r+1})$$

Where λ_{r+1} is the $(r+1)^{th}$ ordered eigenvalue of Π , and T is the sample size. The critical values tabulated by Johansen and Juselius (1990) will be used for these tests.

Vector Error Correction (VEC) Models

A vector error correction (VEC) model is a restricted VAR designed for use with non-stationary series that are known to be cointegrated. The VEC has cointegration relations built into the specification so that it restricts the long-run behavior of the endogenous variables to converge to their cointegrating relationships while allowing for short-run adjustment dynamics. The cointegration term is known as the error correction term since the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments.

When the variables are cointegrated, the corresponding error correction representations must be included in the system. By doing so, one can avoid misspecification and omission of the important constraints. Thus, the VAR in (12) can be re-parameterized as a Vector Error Correction Model (VECM) form: (Hamelton, 1994.)

$$\Delta y_t = \Pi Y_{t-1} + \sum_{i-1}^{p-1} \Gamma_i \Delta y_{t-i} + BX_t + \varepsilon_t$$

where $\Pi = -I_n + \sum_{i-1}^{p} A_i$, $\Gamma_i = -\sum_{j=i+1}^{p} A_j$ and I_n is an identity matrix

The above specification of VECM contains information on both the short and the long run adjustment to changes in y_t via estimating Γ and Π, respectively. Matrix Π can be decomposed as $\Pi=\alpha\beta'$, where α is nxr matrix of speed of adjustments, and β is an nxr matrix of parameters which determines the cointegrating relationships matrix of long- run coefficients such that $\beta' y_{t-k}$ represent the multiple cointegration relationships. The columns of β are interpreted as long-run equilibrium relationships between variables. The matrix α determines the speed of adjustment towards this equilibrium. Values of α close to zero imply slow convergence and r,$0 \le r \le n$ is the rank of the matrix Π and represents the number of cointegrating vectors in the system which can be determined using the Johansen Maximum Likelihood method (Bourbonnais, 2007; Bourbonnais & Terraza, 2008).

Model Checking
A wide range of procedures is available for checking the adequacy of VAR and VECMs. They should be applied before a model is used for specific purpose to ensure that it represents the data adequately.

Test of Residual Autocorrelation
Portmanteau Autocorrelation Test
The portmanteau test for residual autocorrelation (Gujarati, 2004) checks the null hypothesis that all residual auto-covariances are zero, that is, H_0: $E(\varepsilon_t, \varepsilon_{t-i}) = 0$, $i = 1,2, \ldots$. This hypothesis is tested against the alternative that at least one autocovariance and, hence, one autocorrelation is nonzero. The test statistics is based on the residual autocovariances and has the form

$$Q_h = T\sum_{j=1}^{h} tr(\hat{\gamma}'_j \hat{\gamma}_0^{-1} \hat{\gamma}_j \hat{\gamma}_0^{-1}) \qquad (15)$$

$$\hat{\gamma}_j = T^{-1}\sum_{t=j+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-j} \qquad (16)$$

and the $\hat{\varepsilon}_t$'s are the estimated residuals. For unrestricted residuals stationary VAR(p) process, the null distribution of Q_h is approximated by $\chi^2_{k2(h-p)}$ distribution if T and h approaches infinity such that $h/T \to 0$.
Alternatively (especially in small samples), a modified statistic

$$Q^*_h = T^2\sum_{j=1}^{h} \frac{1}{T-j} tr(\hat{\gamma}'_j \hat{\gamma}_0^{-1} \hat{\gamma}_j \hat{\gamma}_0^{-1}) \qquad (17)$$

is usually employed in place of (15)

Autocorrelation LM Test
This test was developed by Breusch and Godfrey in 1978 (Breusch & Godfrey, 1978). Assume a VAR model for the error u_t given by

$$u_t = D_1 u_{t-1} + \ldots + D_h u_{t-h} + v_t \qquad (18)$$

The quantity v_t denotes a white nose error term. Thus, to test autocorrelation in u_t, we test

$$H_0: D1 = \ldots = D_h = 0 \text{ against } H_1: D_j \neq 0 \text{ for at least one } j,\ j = 1, \ldots, h.$$

We use the LaGrange Multiplier method to perform the test. This method is very useful for finding optimal estimates under constraint conditions. Under H_0, we only need to estimate the regular VAR model ($u_t = v_t$). So the constrained case estimates are simple. To determine the test statistic we begin with the auxiliary regression model

$$\hat{U} = BZ + D\hat{U} + E \qquad (19)$$

where $\hat{U} = [\hat{u} \ldots \hat{u}_T]$, $Z = [Z_0 \ldots Z_{T-1}]$ with $Z_t = [1^T\ y^T \ldots y^T_{t-p+1}]^T$ and $D = [D_1 \ldots D_h]$.
Define F_i such that

$$\hat{U}\ F_i \hat{U}^T = \sum_{t=i+1}^{T} \hat{u}_t \hat{u}_{t-I} \qquad (20)$$

with $F = [F_1 \ldots\ldots F_h]$ and $\hat{U} = (I \oplus \hat{U})\ F^T$.
This yields the least squares estimate of D given by

$$\hat{D} = \hat{U}\hat{U}^T[\hat{U}\hat{U}^T - \hat{U}Z^T(ZZ^T)^{-1} Z\hat{U}^T]^{-1} \qquad (21)$$

The standard χ^2 test statistic for testing whether $D = 0$ (no autocorrelation) is, under H_0:

$$\lambda_{LM}(h) = vec(\hat{D})^T ([\hat{U}\hat{U}^T - \hat{U}Z^T(ZZ^T)^{-1} Z\hat{U}^T] \oplus \hat{\Sigma}_u)\, vec(\hat{D}) \quad (22)$$

$$\lambda_{LM}(h) \xrightarrow{d} \chi^2 (hk^2) \qquad (22)$$

Normality of the Residuals
Multivariate generalization of the Jarque-Bera test (Jarque & Bera 1987) has been a major technique to test the multivariate normality of the u_t, the vector of the error terms in a multivariate time series model. This tests the skewness and kurtosis properties of the u_t (3rd and 4th moments) against those of a multivariate normal distribution of the appropriate dimension.

H_0: $E(u^s_t)^3 = 0$ (skewness) and $E(u^s_t)^4 = 3$(kurtosis) against H_1: $E(u^s_t)^3 \neq 0$ and $E(u^s_t)^4 \neq 3$

It is possible that the first four moments of the u_t match the multivariate normal moments, and the u_t are still not normally distributed. It is hoped that most of the "normal" properties desired by the model fitted in the u_t are met by these four moments. This situation has an analog in

linear regression. We assume that the errors are independent, but we can only test whether they are correlated. In linear regression, it is adequate to test the correlation of the residuals. If they are uncorrelated, that is enough "independence" for getting the variance calculations correct. We don't worry about the other forms of dependence.
Formulation of the Jarque-Bera test uses a mean adjusted form of the VAR (p) model

$$\hat{u} = (y_t - \bar{y}) - \hat{A}_1 (y_{t-1} - \bar{y}) - \ldots - \hat{A}_p (y_{t-p} - \bar{y}) \quad (23)$$

with $\widetilde{var}(u) = \frac{1}{T-kp-1} \sum_{t=1}^{T} \hat{u}_t \hat{u}_t^T$

Let $\hat{P}$ be the matrix satisfying $\hat{P}\hat{P}^T = \hat{\Sigma}_u$ such that $lim(\hat{P} - P) = 0$
Now we define the standardized residuals and their sample moments

$$\hat{w}_t = \hat{P}^{-1}\hat{u}_t$$

$$\hat{b}_1 = (\hat{b}_{11} \ldots \hat{b}_{k1}) \ni \hat{b}_{i1} = \frac{1}{T}\sum_{t=1}^{T} w^3_{it}$$

$$\hat{b}_2 = (\hat{b}_{12} \ldots \hat{b}_{k2}) \ni \hat{b}_{i2} = \frac{1}{T}\sum_{t=1}^{T} w^4_{it}$$

Finally our test statistics are

$$\lambda_S = T\hat{b}_1^T\hat{b}_1 / 6$$

$$\lambda_S = T(\hat{b}_2^T - 3 1)^T (\hat{b}_2^T - 3 1) / 24$$

$$\lambda_{SK} = \lambda_S + \lambda_K \quad (24)$$

The third and fourth moment of u_t should be 0 and 3
Under the third moment assumption

$$\lambda_S \xrightarrow{d} \chi^2(k) \quad (25)$$

Under the moment assumption

$$\lambda_K \xrightarrow{d} \chi^2(k) \quad (26)$$

Under both assumptions

$$\lambda_{SK} \xrightarrow{d} \chi^2(2k) \quad (27)$$

Forecasting

Forecasting is one of the main objectives of multivariate time series analysis. Forecasting from a VAR model is similar to forecasting from a univariate models.
Consider first the problem of forecasting future values of Y_t when the parameters Π of the VAR (p) process are assumed to be known and there are no deterministic terms or exogenous variables. The best linear predictor, in terms of minimum mean squared error (MSE), of Y_{t+1} or 1-step forecast based on information available at time T is

$$Y_{T+1|T} = c + \Pi_1 Y_T + \Pi_2 Y_{T-1} + \cdots + \Pi_p Y_{T-p+1} \quad (28)$$

for T ≥ p
Forecasts for longer horizons *h* (*h*-step forecasts) can be obtained using the chain-rule of forecasting as

$$Y_{T+h|T} = c + \Pi_1 Y_{T+h-1|T} + \cdots + \Pi_p Y_{T+h-p|T} \quad (29)$$

where $Y_{T+j|T} = Y_{T+j}$ for $j \leq 0$. The *h*-step forecast errors may be expressed as

$$Y_{T+h} - Y_{T+h|T} = \sum_{s=0}^{h-1} \Psi_s \varepsilon_{T+h-s} \quad (30)$$

where the matrices Ψ_s are determined by recursive substitution

$$\Psi_s = \sum_{j=1}^{p-1} \Psi_{s-j} \Pi_j \quad (31)$$

wth $\Psi_0 = I_n$ and $\Pi_j = 0$ for j > p. The forecasts are unbiased since all of the forecast errors have expectation zero, and the MSE matrix for $Y_{t+h|T}$ is

$$\Sigma(h) = MSE(Y_{T+h} - Y_{T+h|T} = \sum_{s=0}^{h-1} \Psi_s \Sigma \Psi_s \quad (32)$$

Now consider forecasting YT+h when the parameters of the VAR(p) process are estimated using multivariate least squares. The best linear predictor of Y_{T+h} is now

$$\hat{Y}_{T+h|T} = c + \hat{\Pi}_1 \hat{Y}_{T+h-1|T} + \cdots + \hat{\Pi}_p \hat{Y}_{T+h-p|T} \quad (33)$$

where $\hat{\Pi}_j$ are the estimated parameters matrices. The h-step forecast error is given by

$$Y_{T+h} - \hat{Y}_{T+h|T} = \sum_{s=0}^{h-1} \Psi_s \varepsilon_{T+h-s} + (Y_{T+h} - \hat{Y}_{T+h|T})$$

and the term (YT+h − ŶT+h|T) captures the part of the forecast error due to estimating the parameters of the VAR. The MSE matrix of the h-step forecast is then

$$\hat{\Sigma}(h) = \Sigma(h) + MSE(Y_{T+h} - Y_{T+h|T}) \quad (34)$$

In practice, the second term MSE ($Y_{T+h} - \hat{Y}_{T+h|T}$) is often ignored and $\hat{\Sigma}(h)$ is computed using (33) as

$$\hat{\Sigma}(h) = \sum_{s=0}^{h-1} \hat{\Psi}_s \hat{\Sigma} \hat{\Psi}_s \quad (35)$$

where $\hat{\Psi}_s = \sum_{j-1}^{s} \hat{\Psi}_s - \hat{\Pi}_j$.

Lütkepohl (1991) gives an approximation to MSE ($Y_{T+h} - \hat{Y}_{T+h|T}$) which may be interpreted as a finite sample correction to (35).

Measures of forecasting accuracy

In most forecasting situations, accuracy is treated as the overriding criterion for selecting a forecasting method. In many instances, the word "accuracy" refers to the goodness of fit, which simply refers to how well the forecasting model is able to reproduce the data that are already known. To the consumer of forecasts, it is the accuracy of the future forecast that is most important.

If Y_t is the actual observation for the period t and F_t is the forecast for the sample period, then the error is defined as

$$v_t = Y_t - F_t \quad (36)$$

Usually, F_t is calculated using data $Y_1, \ldots, Y_{t-1}$. It is a one-step forecast because it is forecasting one period ahead of the last observation used in the calculation .Therefore, we describe v_t as a one-step forecast error. It is the difference between the observation Y_t and forecast made using all observations up to but not including Y_t.

If there are observations and forecasts for T time periods, then there will be n error terms, and the following standard statistical measures can be defined:

$$\text{Mean Error } (ME) = \frac{1}{T}\sum_{t=1}^{T} v_t$$

$$\text{Mean absolute Error } (MAE) = \frac{1}{T}\sum_{t=1}^{T} |v_t|$$

$$\text{Mean Squared Error } (MSE) = \frac{1}{T}\sum_{t=1}^{T} v_t^2$$

Equation (36) can be used to compute the error for each period. These can then be averaged as given above to give the mean error (ME). However, the ME is likely to be small since positive and negative errors tend to offset one another. In fact, the ME will only tell you if there is systematic under-or over forecasting, called the forecasting bias. It does not give much indication as to the size of the typical errors.

Therefore, the mean absolute error (MAE) is defined by first making error positive by taking its absolute value, and then averaging the results. The idea behind the definition of mean square error (MSE) is similar. Here the errors are made positive by squaring each one, and then the squared errors are averaged. The MSE has advantage of being more interpretable and is easier to explain to non-specialist.

Each of these statistics deals with measures of accuracy whose size depends on the scale of the data. Therefore, they do not facilitate comparison across different time series and for different time intervals. To make comparisons, we need to work with relative or percentage error measures given by

$$PE_t = \left(\frac{Y_t - F_t}{Y_t}\right) \times 100 \quad (37)$$

Arising from (37), the following two relative measures are frequently used:

$$\text{Mean percentage Error } (MPE) = \frac{1}{T}\sum_{t=1}^{T} PE_t \quad (38)$$

$$\text{Mean percentage Absolute Error } (MPAE) = \frac{1}{T}\sum_{t=1}^{T} |PE_t| \quad (39)$$

Equation (37) can be used to compute the percentage error for any time period .These can be averaged as in equation (38) to give the mean percentage error. However, as with the ME, the MPE is likely to be small since positive and negative PEs tends to offset one another. Hence the MAPE is defined using absolute values of PE in equation (39).

Alternatively, Theil's U statistic can be used as a measure of forecasting accuracy. Like MAPE statistics, high values suggest poor performance in the forecast. However, and unlike MAPE, the U-Theil corrects the performance scale that MPAE had. Theil's U can be estimated as:

$$\frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(Y_t - F_t)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(F_t)^2}+\sqrt{\frac{1}{n}\sum_{t=1}^{n}(Y_t)^2}} \qquad (40)$$

The scaling of U is such that it will always lie between 0 and 1. If U = 0,Yt=Ft for all forecasts and there is a perfect fit; if U = 1 the predictive performance is not good.

Structural Vector Autoregressive (SVAR) Analysis

The general VAR (p) model has many parameters, and they may be difficult to interpret due to complex interactions and feedback between the variables in the model. As a result, the dynamic properties of a VAR (p) are often summarized using various types of structural analysis. One of the main types of structural analysis summaries is the Granger causality tests.

Analysis and Results

The time series under consideration were checked for stationary before any suitable time series model could be considered for the data. That is, variables have to be tested for the presence of unit root(s) thereby the order of integration of each series is determined. Figs1 and 2 suggests that the series of the endogenous variables display non-stationary behaviours.

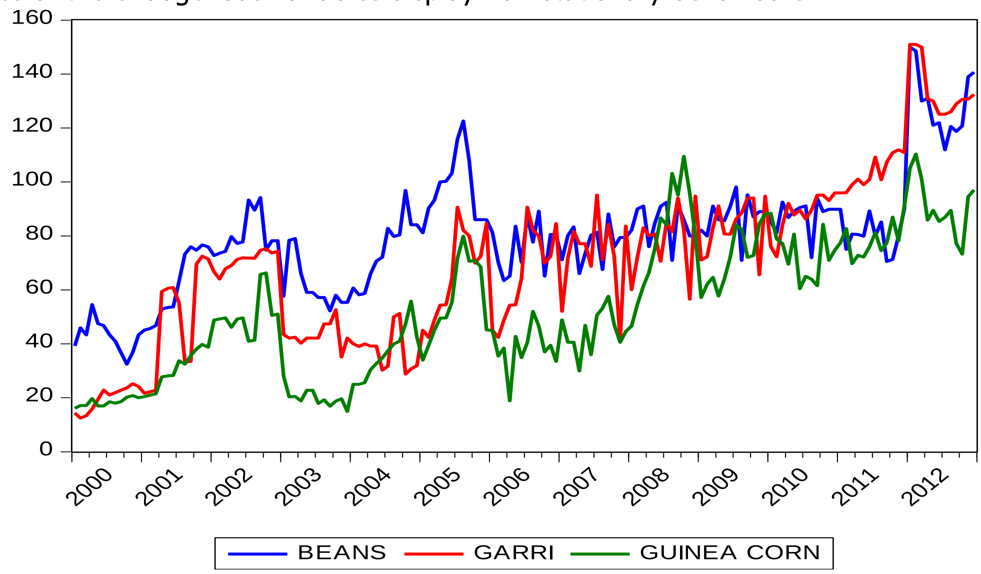

Fig. 1: Time plots of the prices of beans, Garri and Guinea corn for twelve years from 2000 to 2012.

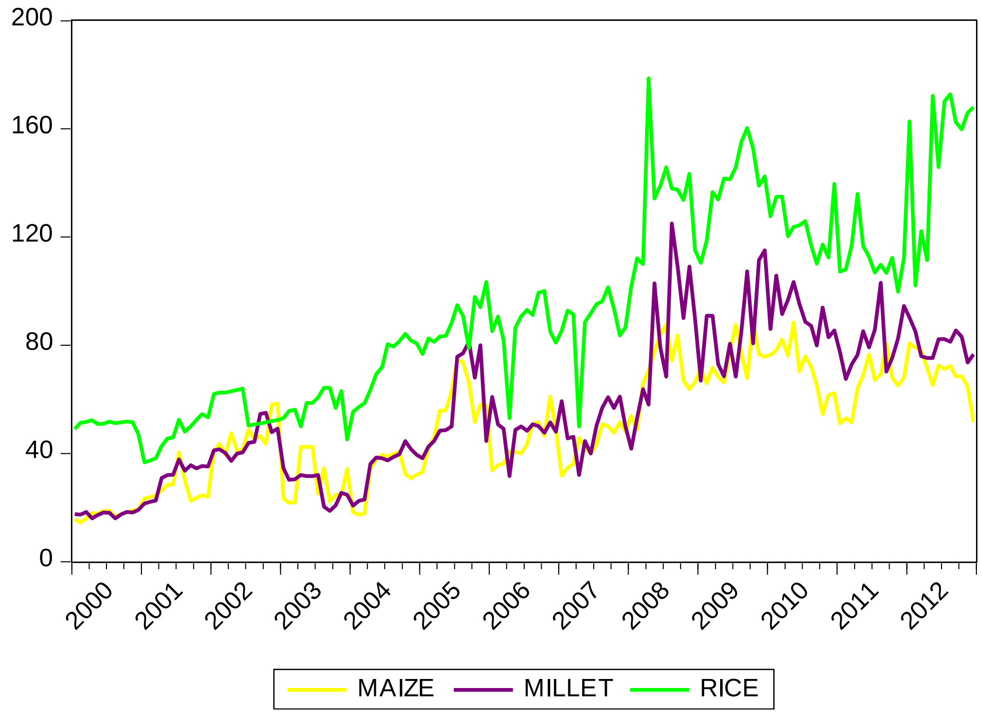

Fig. 2: Time plots of the prices of maize, Millet and Rice for twelve consecutive years from 2000 to 2012.

Hypothesis

H_0: The series are non stationary ($\rho=1$) or $\alpha = 0$ Vs. H_1: The series are stationary ($\rho<1$) or $\alpha < 0$

Table 1: Table of results for stationarity of the series using the Unit root test

Series	Level with intercept		Level with intercept and Trend	
	Test statistic	Prob.*	Test statistic	Prob.*
	ADF	ADF	ADF	ADF
Maize	-2.812085	0.0589	-3.302502	0.0641
Millet	-1.952513	0.3077	-3.280733	0.0734
Rice	-1.107322	0.7123	-3.121823	0.7237
Guinea-Corn	-2.101532	0.2445	-3.091644	0.0508
Beans	-1.864203	0.3485	-3.174035	0.0962
Garri	-2.80328	0.0601	-3.062203	0.0817
Critical value (5%)	-2.880211		-3.439075	

Test results, presented in Table 1, indicated that the null hypothesis that the series in levels contain unit root could not be rejected for all the six series. That is, the respective p-values are greater than significance levels $\alpha = 0.05$, hence the series are integrated of degree one (I(1)). Since the null hypothesis cannot be rejected, in order to determine the order of integration of the non-stationary time series, the same tests were applied to their first differences (Figs. 3 and 4). The order of integration is the number of unit roots that is differenced in the series so as to be stationary.

Table 3: Unit root test results (after first difference)

Series	Level with Intercept		Level with Intercept and Trend	
	Test statistic	Prob.*	Test statistic	Prob.*
	ADF	ADF	ADF	ADF

Maize	-14.32589	< 0.0001	-11.27824	<0.0001
Millet	-13.14097	<0.0001	-12.13143	<0.0001
Rice	-19.85791	<0.0001	-19.81501	<0.0001
Guinea-Corn	-14.13909	<0.0001	-12.12869	<0.0001
Beans	-16.28760	<0.0001	-16.26299	<0.0001
Garri	-13.02529	<0.0001	-12.97029	<0.0001
Critical value (5%)	-2.880211		-3.439075	

The results in Table 3 indicate that the null hypothesis is rejected for the first differences of the six staple food stuffs given that p-values less than 5% level of significance. This implies that the six time series are integrated of degree one (I(1)).

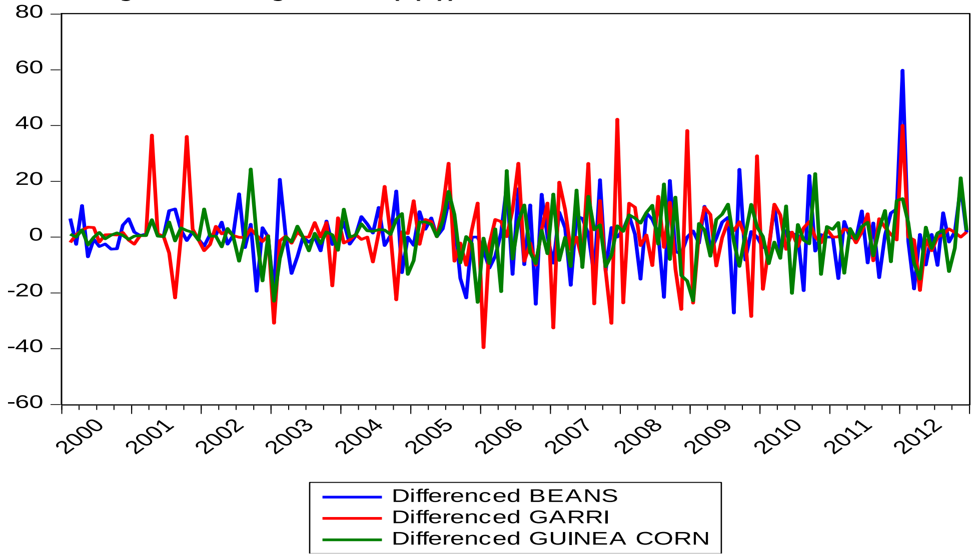

Fig. 3: Time plot of differenced data on prices of Beans, Garri and Guinea Corn

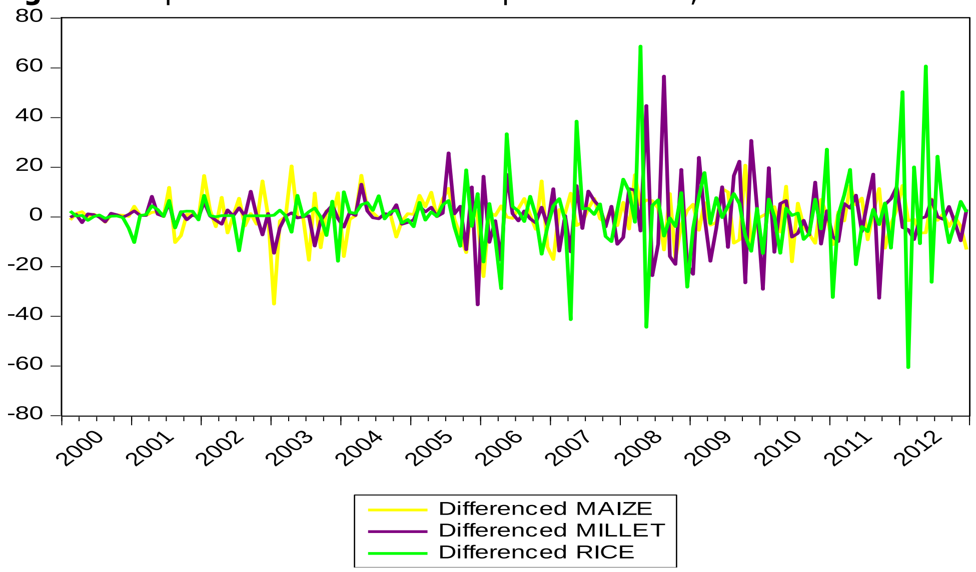

Fig. 5: Time plot of differenced data on prices of Maize, Millet and Rice

Var Model Specification

Estimating for Order of the VAR

Specifying the lag length has strong implications for subsequent modeling choices. Choosing too few lags could lead to systematic variation in the residuals whereas if too many lags are chosen it comes with the penalty of fewer degrees of freedom (as adding another lag, to the p x p variables). To determine the optimum lag length, we test for statistics which include: Final

Prediction Error (FPE), Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and Hannan Quin Information Criterion (HQIC) are diverse. The FPE and AIC and HQIC indicate lag length of two. We therefore choose lag length of two.

Table 4: VAR lag order selection results

Lag	FPE	AIC	HQIC	SIC
0	1.33e+14	49.5502	49.59895	49.67013
1	1.22e+12	44.85826	46.26747	49.67013
2	***1.05e+12**	***44.70887**	***45.19920**	***46.26747**
3	1.20e+12	44.82961	47.10755	47.10755
4	2.20e+13	44.94439	47.94169	47.94169

***indicates lag order selected by the criterion**

Based on the results in Table 4 using the computed FPE, AIC, HQIC and SIC values, it can be concluded that the optimum lag length is 2.

Cointegration Analysis

Since the variables are integrated of order one, we proceed to test for co-integration. Johansen (1995) cointegration test is applied at the predetermined lag 2. In these tests, Maximum Eigenvalue statistic or Trace statistic is compared to special critical values. The maximum eigenvalue and trace tests proceed sequentially from the first hypothesis –no cointegration– to an increasing number of cointegrating vectors.

The results of Cointegration tests for Rice, maize, gari, millet, guinea corn, palm oil and beans are reported in Table 4.5. The trace statistic indicates that at least two cointegrating vector ($r \geq 1$) exists in the system at the 95 percent confidence level (estimated LR statistic, 148.70> 125.62 and 97.52>95.75, 5 per cent critical value). In order to cross check for identifying the specific number of cointegrating vectors, the maximal eigenvalue statistic is further employed. This statistic reduced the cointegrating equation to only one cointegrating relationship at the 95 per cent confidence level in this system (estimated LR statistic, 51.17 > 46.23, 95 per cent critical value).

Table 5: Johansen Cointegration test results (by assumption of Linear deterministic trend)

Number of cointegrating vector	Eigenvalue	Trace Test			Maximum Eigenvalue Test		
		Statistic	5% critical value	Prob.**	statistic	5% critical value	Prob.**
None *	0.272467	111.8006	95.75366	0.0025	48.03246	40.07757	0.0052
At most 1	0.172277	63.76814	69.81889	0.1382	28.55064	33.87687	0.1893
At most 2	0.116442	35.21750	47.85613	0.4365	18.69363	27.58434	0.4386
At most 3	0.069094	16.52386	29.79707	0.6749	10.81118	21.13162	0.6661

Table 6: Table of normalized cointegrating coefficients and their respective standard errors

Normalized cointegrating coefficients (standard error in parentheses)

BEANS	GARRI	G.CORN	MAIZE	MILLET	RICE
1.000000	0.009934	-1.488467	-7.708178	7.595088	-0.766970
	(0.41205)	(0.88302)	(1.22142)	(1.11646)	(0.39983)

From the Johansen co-integration test, it was discovered that the rank of co-integration matrix to be equal to one. Consequently, the co-integrating vector is given by β= (1, 0.009934, -1.488467, -7.708178, 7.595088, -0.766970)

The values correspond to the co-integrating coefficient of BEANS (normalized to one), GARRI, GUINEA CORN, MAIZE, MILLET, PALM OIL and RICE respectively. Thus, the vector above can be expressed as follows;

$$Beans_t = -0.009934Garri_t + 1.4884672G.corn_t + 7.708178Maize_t - 7.595088Millet_t + 0.766970Rice_t$$

Vector Error Correction Model Estimation

Having concluded that variables in the VAR model appeared to be cointegrated, we proceed to estimate the short run behavior and the adjustment to the long run models, which is represented by VECM. The VEC model has the following structure:

$$\Delta Y_t = \mu + \sum_{i=1}^{p} \Gamma_i \Delta Y_{t-i} + \alpha BX_{t-1} + \varepsilon_t ,$$

where BX_t is the error correction term given by $\beta' \mathrm{Y}_t$ and β is the cointegrating vector. The responses of BEANS, GARRI, GUINEA-CORN, MAIZE, MILLET and RICE to short-term output movements are captured by the Γ_i coefficient matrices. The coefficient vector α reveals the speed of adjustment to the equilibrium which measures the deviation from the long-run relationship between the series.

The long run equation is given as follows:

$$Beans_t = 5.864122 - 0.009934Garri_t + 1.4884672G.corn_t + 7.708178Maize_t - 7.595088Millet_t + 0.766970Rice_t$$

(**)

The value 0.0099 suggests that a one unit increase in the average price of Garri induces, on average, an decrease of about 0.01 units in the average price of Beans when the price of other staple foods are held constant. Similarly, one unit increase in the average price of G.corn leads to an increase of about 1.49 units in the average price of Beans when the price of other staple foods are held constant. Also one unit increase in the average price of Maize leads to an increase of about 7.71 units in the average price of Beans when the price of other staple foods are held constant. Furthermore, one unit increase in the average price of Millet will lead to about 7.60 units decrease in the average price of Beans when the price of other staple foods are held constant. Finally, one unit increase in the average price of Rice will lead to about 0.767 units increase in the average price of Beans when the price of other staple foods are held constant.

From the VAR model (**) the following Vector Error Correction Models are estimated:

Model of Average Price of Beans:

$$\Delta BnsP_t = 0.001 * (BnsP_{t-1} - 0.0099Gri_t + 1.49G.c_t - 7.71Maz_t - 7.60Mlt_t + 0.77Rce_t + 5.8) - 0.30 * \Delta BnsP_{t-1} - 0.02 * \Delta BnsP_{t-2} - 0.01 * \Delta GriP_{t-1} + 0.05 * \Delta GriP_{t-2} - 0.12 * \Delta G.cP_{t-1} - 0.01 * \Delta G.cP_{t-2} + 0.02 * \Delta MazP_{t-1} - 0.002\Delta MazP_{t-2} + 0.23\Delta MltP_{t-1} - 0.09\Delta MltP_{t-2} + 0.08\Delta RceP_{t-1} - 0.05\Delta RceP_{t-2} + 0.69$$

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Model of Average Price of Garri:

$$\Delta GriP_t = 0.02 * (BnsP_{t-1} - 0.0099Gri_t + 1.49G.c_t - 7.71Maz_t - 7.60Mlt_t + 0.77Rce_t + 5.8) - 0.08 * \Delta BnsP_{t-1} - 0.03 * \Delta BnsP_{t-2} - 0.42 * \Delta GriP_{t-1} + 0.17 * \Delta GriP_{t-2} - 0.13 * \Delta G.cP_{t-1} - 0.34 * \Delta G.cP_{t-2} + 0.24 * \Delta MazP_{t-1} - 0.13\Delta MazP_{t-2} + 0.12\Delta MltP_{t-1} - 0.14\Delta MltP_{t-2} + 0.08\Delta RceP_{t-1} + 0.02\Delta RceP_{t-2} + 0.91$$

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Model of Average Price of Guinea Corn:

$$\Delta G.cP_t = -0.04 * (BnsP_{t-1} - 0.0099Gri_t + 1.49G.c_t - 7.71Maz_t - 7.60Mlt_t + 0.77Rce_t + 5.8) + 0.22 * \Delta BnsP_{t-1} + 0.11 * \Delta BnsP_{t-2} + 0.02 * \Delta GriP_{t-1} + 0.01 * \Delta GriP_{t-2} - 0.27 * \Delta G.cP_{t-1} - 0.06 * \Delta G.cP_{t-2} - 0.04 * \Delta MazP_{t-1} - 0.02\Delta MazP_{t-2} + 0.15\Delta MltP_{t-1} - 0.11\Delta MltP_{t-2} - 0.03\Delta RceP_{t-1} + 0.002\Delta RceP_{t-2} + 0.41$$

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Model of Average Price of Maize:

$$\Delta MazP_t = 0.06 * (BnsP_{t-1} - 0.0099Gri_t + 1.49G.c_t - 7.71Maz_t - 7.60Mlt_t + 0.77Rce_t + 5.8) - 0.22 * \Delta BnsP_{t-1} - 0.06 * \Delta BnsP_{t-2} - 0.05 * \Delta GriP_{t-1} + 0.03 * \Delta GriP_{t-2} - 0.05 * \Delta G.cP_{t-1} + 0.01 * \Delta G.cP_{t-2} + 0.12 * \Delta MazP_{t-1} + 0.04\Delta MazP_{t-2} + 0.09\Delta MltP_{t-1} - 0.02\Delta MltP_{t-2} + 0.10\Delta RceP_{t-1} + 0.06\Delta RceP_{t-2} + 0.02$$

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Model of Average Price of Millet:

$$\Delta MltP_t = -0.10 * (BnsP_{t-1} - 0.0099Gri_t + 1.49G.c_t - 7.71Maz_t - 7.60Mlt_t + 0.77Rce_t + 5.8) + 0.01 * \Delta BnsP_{t-1} + 0.15 * \Delta BnsP_{t-2} - 0.02 * \Delta GriP_{t-1} + 0.09 * \Delta GriP_{t-2} - 0.03 * \Delta G.cP_{t-1} + 0.07 * \Delta G.cP_{t-2} - 0.23 * \Delta MazP_{t-1} - 0.09\Delta MazP_{t-2} - 0.22\Delta MltP_{t-1} - 0.28\Delta MltP_{t-2} + 0.17\Delta RceP_{t-1} + 0.04\Delta RceP_{t-2} + 0.36$$

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Model of Average Price of Rice:

$$\Delta RceP_t = -0.06 * (BnsP_{t-1} - 0.0099Gri_t + 1.49G.c_t - 7.71Maz_t - 7.60Mlt_t + 0.77Rce_t + 5.8) - 0.04 * \Delta BnsP_{t-1} - 0.07 * \Delta BnsP_{t-2} - 0.07 * \Delta GriP_{t-1} + 0.04 * \Delta GriP_{t-2} + 0.03 * \Delta G.cP_{t-1} + 0.01 * \Delta G.cP_{t-2} - 0.19 * \Delta MazP_{t-1} - 0.26\Delta MazP_{t-2} + 0.25\Delta MltP_{t-1} + 0.08\Delta MltP_{t-2} - 0.53\Delta RceP_{t-1} - 0.14\Delta RceP_{t-2} + 1.36$$

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where: 'Δ' stands for first difference (D), the value in the bracket is the error correction term and the coefficients of error correction term are called adjustment coefficients.

Model Diagnosis

In order to ascertain whether the model provides an appropriate representation, a test for misspecification is performed.

Test for Residual Autocorrelation

Portmanteau Q-statistic test for VAR model residual serial correlation is presented below. This test is used to test for the overall significance of the residual autocorrelations up to lag 2.

Table 7: Test for residual autocorrelation and serial correlation

Lag	Q-stat	Prob.	Adj Q-Stat	LM-Stat	Prob
1	9.059484	NA*	7.916442	61.94690	0.2472
2	40.80412	NA*	40.97879	63.43891	0.0911
3	96.48770	0.4447	96.9577	61.94073	0.2311

H_0: No residual autocorrelation against H_1: There is residual autocorrelation

Since p-value (0.4447) > α =(0.05), we cannot reject H_0. Hence we conclude that there is no residual autocorrelation at lag 3 and it is white noise.

Testing For Normality of Residual

Multivariate version of the Jarque Bera tests is used to test the normality of the residuals. It compares the 3rd and 4th moments (Skewness and Kurtosis) to those from a normal distribution.

The test has null hypothesis indicating that the error term in the model has skewness and kurtosis corresponding to a normal distribution.

H_0: $E(u^s_t)^3 = 0$ (skewness) and $E(u^s_t)^4 = 3$(kurtosis) Vs. H_1: $E(u^s_t)^3 \neq 0$ and $E(u^s_t)^4 \neq 3$

The results in Table 8 show that the null hypothesis was not rejected

Table 8: Table of results of tests for normality of residual

SERIES	Skewness	Kurtosis	Jarque bera	Prob.
Maize	0.093690	2.848694	2.186625	0.910909
Millet	0.335336	2.708065	2.950469	0.621739

Rice	0.234704	2.870322	3.795935	0.149873
G.corn	0.149288	2.999581	1.399126	0.998004
Beans	0.112439	2.755469	2.50107	0.158121
Garri	0.236493	2.845892	1.608529	0.447417

The values of the Skewness showed that the curve of the staple foodstuffs are centered since they are approximately zero and not also have an abnormal peak with kurtosis values of approximately three also the Jarque-bera normality test shows that the price for all the six staple foodstuffs is normally distributed since the null hypothesis that the process is normally distributed was not rejected at 0.05 critical level.

Forecasting

One of the fundamental applications of time series analysis or developing a time series model is forecasting. The previous discussion confirm that vector error correction model of order two is a good model to describe the series. In this section we examine the forecasting accuracy of the fitted model and then make a forecast for January 2013 to December 2015.

Evaluation of Accuracy

The mean square error (MSE), root mean square error (RMSE), mean absolute error (MAE) and Theil U statistics were used to assess the forecasting performance. The RMSE and MAE statistics are scale-dependent measures, but allow a comparison between the actual and forecast values. The Theil-U statistics is independent of the scale of the variables and is constructed to lie between zero and one, zero indicating a perfect fit. In evaluating the performance of the forecasting models, the lower the RMSE, MAE, MAPE and Theil-U statistic, the better the forecasting accuracy.

Table 9 reports the forecasting accuracy statistics of the estimated model. The result indicates that the estimated models are good enough to describe the series. The RMSE, MAE, MAPE values are less than 5% and Theil-U statistics is close to zero, which indicates the difference between the actual value and the predicted value is very small. That is, the predictive powers of the models are better and suitable for n-step ahead forecasting.

Table 9: Forecasting Accuracy statistic

Forecast sample: January 2000 to December 2012

Variables	RMSE	MAE	MAPE	Theil-U
Maize	4.01	3.91	5.01	0.000559
Millet	3.77	4.79	3.78	0.000089
Rice	3.97	3.12	4.19	0.000661
G.corn	4.89	3.71	1.09	0.000078
Beans	3.54	4.73	2.11	0.000111
Garri	2.99	4.02	4.18	0.000052

Post Forecasting Analysis

Post forecasted values for the beans and the other staple foodstuffs from January 2013 to December 2015, using the vector error correction model are presented in the Table 10.

Table 10: Forecast from January 2013 to December 2015

Time	Beans	Rice	Maize	Millet	Guinea-Corn	Garri
2013m1	140.7914	166.0415	55.1923	71.4898	94.6616	132.0118
2013m2	140.8526	166.3924	56.4112	67.0044	94.5677	133.3475
2013m3	140.8515	166.7432	56.1598	67.5915	94.8838	133.1017
2013m4	141.6500	167.7597	56.5383	67.7932	95.2179	134.1484
2013m5	142.2446	168.2834	56.8742	67.9387	95.7304	134.8942

2013m6	142.8143	169.1100	57.1020	68.2607	96.2223	135.6213
2013m7	143.4502	169.8240	57.3707	68.6242	96.7052	136.3909
2013m8	144.0659	170.5790	57.6431	68.9321	97.2067	137.1630
2013m9	144.6799	171.3162	57.9043	69.2733	97.7015	137.9208
2013m10	145.2985	172.0650	58.1701	69.6065	98.1977	138.6874
2013m11	145.9157	172.8068	58.4355	69.9406	98.6940	139.4516
2013m12	146.5327	173.5522	58.7005	70.2744	99.1904	140.2159
2014m1	147.1501	174.296	58.9656	70.6090	99.6866	140.9803
2014m2	147.7673	175.0405	59.2308	70.9430	100.1830	141.7448
2014m3	148.3846	175.7847	59.4959	71.2772	100.6793	142.5091
2014m4	149.0018	176.5290	59.7610	71.6114	101.1756	143.2736
2014m5	149.6191	177.2733	60.0261	71.9456	101.6720	144.0380
2014m6	150.2363	178.0176	60.2912	72.2798	102.1683	144.8024
2014m7	150.8536	178.7619	60.5563	72.6140	102.6646	145.5668
2014m8	151.4709	179.5062	60.8215	72.9482	103.1609	146.3312
2014m9	152.0881	180.2505	61.0866	73.2823	103.6572	147.0957
2014m10	152.7054	180.9948	61.3517	73.6166	104.1536	147.8601
2014m11	153.3226	181.7390	61.6168	73.9507	104.6499	148.6245
2014m12	153.9399	182.4833	61.8820	74.2849	105.1462	149.3889

Conclusion

Over the time period considered, all the six series showed an increasing pattern, with non-stationarity structures. In order to examine the VAR model, the unit root tests (ADF tests), identification of the number of lags and cointegration analyses were conducted. Unit root tests indicated that all the series were non-stationary and were stationary at first difference at 5% significant level. The Johansen cointegration test suggests that there was at least one cointegration vector, which describes the long run relationship between the prices of Rice, Maize, Garri, Millet, Guinea-Corn, and Beans in Nigeria.

Finally, forecasting is made using vector error correction (VEC) model. Several model's diagnosis measures indicated that the estimated model is good enough to describe the data. Therefore, post forecasts were made for Beans, Rice, Maize, Millet, Guinea-Corn, and Garri from January 2013 to 2014. The result indicated that the prices of all the staple foodstuffs in Nigeria have a steady increment over time.

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