ANALYSIS OF MISCONCEPTIONS IN ALGEBRAIC EXPRESSION AMONG SENIOR SECONDARY SCHOOL STUDENTS OF DIFFERENT ABILITY LEVELS IN KATSINA STATE

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Abstract
The study employed the content analysis procedures to analyze major misconceptions of different ability students in algebraic expression at senior secondary school level. Sixty (60) senior secondary school III students from the three senatorial districts in Katsina State were stratified and randomly sampled. The researcher used Students Mathematical Test (SMAT) to group the students into different ability levels- high, medium and low and Students' Algebraic Expression Misconception Test (SAEMT) to explore the students' misconceptions in the algebraic expression concepts. Contingency table and Cramer's V were used to answer the research questions while Chi-square test of independence was used to test the statistical significance of the study. The findings showed that invalid/insufficient premise for algebraic manipulations, deficient belief about algebraic structure and restructuring, value mind fixed answers and specific misconceptions categories are found to be the major misconceptions of students of all ability level. The result further indicates these misconceptions significantly related to (a) students' ability with small effect size $\chi^2(df= 6, p< 0.005)= 21.621$; $V= 0.20$ *and (b) error distribution in algebraic expression concepts with small effect size* $\chi^2(df= 6, p< 0.001)= 26.888$; $V= 0.21$ *respectively. The study recommended among others that mathematics teachers, in respective of the ability of the students in mathematics, should give equal attention and treatment to their students' difficulties especially in algebra classes.*

Keywords: Algebra, Algebraic Expression, Misconception, Error, Ability levels

Introduction
Algebra is a cardinal concept in the school mathematics curriculum. Many historically developed algebraic concepts can be observed in the current secondary school algebra curricula throughout the world (Egodawatte, 2011). Algebra, as described by Adeniji and Ibrahim (2015), is a branch of mathematics in which symbols (usually letters of alphabet) represent numbers or numbers of a specified set and are used to represent quantities and to express general relationship that hold for all numbers of a set. This description further explains why its inclusion in the school curricular becomes imperative and how various structural features are connected together to form broader conceptions within algebra in conformity with its historical development.

Wiki answers (2012) listed some of the uses of Algebra as many companies' tools to figure out their annual expenditures, to predict the demand of a particular product and subsequently place their orders for various stores. Algebra expression and equation serve as models for interpreting and making inferences about data.

Following all these points about algebra, it is obvious that students overall mathematics achievements and its subsequent applications in the today's world activities largely depend on algebraic concepts proficiency. However, many efforts to improve Students' algebraic competency have not translated to greater achievement especially at senior Secondary School level. Many are even discontinuing their study of higher-level mathematics because of their lack of success in algebra (Egodawatte, 2011). As result of this, it becomes imperative on the researcher in the field of teaching and learning to study the possible misconceptions students may have in their algebraic classes.

Obviously, there are many factors responsible for students' misconceptions in algebra especially in the algebraic expression concepts, but lack of robust support from researchers in the field of teaching and learning are still noticeable (Seng, 2010). Algebraic expressions contain variables (letters), constant and operational signs. Hence, there is a need for students to understand algebraic structures and the meaning of letters at different situations for them to solve them correctly. According to Bodin and Capponi (1996), the concepts of variables vary with the situation of the problem that students have to handle. This idea of a variable is not only to represent a particular value but can also represent different values at different situation further compound the students' difficulties in handling algebraic expression related problem.

Many conception and misconception have been identified in the literature regarding algebraic expression. Egodawatte (2011), in order to put it in its real perspective, further categorized algebraic expression as expression simplification, evaluation comparing, building and equivalence. This categorization presents opportunity to students to have clear understanding of algebraic expression structures and to mathematics educators and researchers to deeply investigate the possible misconceptions student may have/hold in their algebraic lessons. Those misconceptions identified in the literature include Hallangan's (2006) comment on a teacher model in which students were asked to visually represent an algebraic expression given in four different forms. The same expression was given in four different forms: $4(s+1)$, $s + s + s + s + 4$, $2s + 2(s + 2)$, $4(s + 2) - 4$. A square pool with measurements $s \times s$ and a small square with measurements 1×1 were given as pictures to illustrate the border of a square pool in four different ways related to the above four expressions. There were four main conclusions. First, transition from arithmetic to algebra takes time for students. Second, students preferred numerical answers and to conjoin algebraic terms. Third, on a positive note, visual representations helped students to understand the algorithms in algebra. Fourth, students could not understand the concept of a variable clearly.

Conjoining letters in algebra is to connect together the letters meaninglessly. Researchers (e.g Tall & Thomas, 1991; Stacey & MacGregor, 1999;Egodawatte, 2011) have differences of opinions about reasons for this error. Due to similar meanings of 'and' and 'plus' in natural language, students may consider ab to mean the same as $a + b$ (Tall & Thomas, 1991; Stacey & MacGregor, 1999). Students may erroneously draw on previous learning from other subjects that do not differentiate between conjoining and adding. For example, in chemistry, adding oxygen to carbon produces CO_2 (Egodawatte, 2011).

Kychemann in Seng (2010) also descried some misconceptions which involved students' reluctance in accepting algebraic statement as an answer instead they tend to give numerical values. In addition, student errors were also noticed when expanding bracket with negative signs. This type of error occurs as a result of students' insufficient knowledge of distributive properties of algebra (Ayres, 2000; Sakpakorn & Haries, 2003; Adeniji & Ibrahim, 2015). Understanding of basic arithmetic operation and high manipulating skills are basic criteria for the success in acquisition of algebraic knowledge (Warren, 2003; Adeniji & Ibrahim, 2015). Lack of the structural features of algebra causes many types of misconceptions student have when solving algebraic expression related problems. Misconception is referred to as logically persistent and rule based errors committed by students when solving mathematical problems. Because students have logically constructed their misconceptions from their experiences, they are very attached to them, convincingly belief in them and find it very difficult to give them up.Thus, there is an urgent need to analyse the possible misconceptions of students of different ability levels may have when given algebraic expression related problems.

Various approaches are being used by mathematics educators and teachers to enhance students' mathematics learning, among them is ability grouping approach. The practice according to rationalists' ideal facilitates teaching and learning by enhancing both teachers and students' adjustment. This is because if those in the medium and low ability group are identified, teachers and school system can give more attention and provide intervention program for them. But studies in the area of students' error and misconceptions have shown that even students considered as higher achiever can also hold some unexpected mathematical misconceptions. For instance, Kirshner (1985) confirmed that over-generalization of rules is common in almost every student even successful one before achieving fluency in manipulation skills. Thus, assessing students' errors and misconceptions in mathematics, especially in algebra, is highly inclusive and worthwhile.

Basic algebraic concepts in secondary school algebra are closely linked. If we could study those concepts together in a study and examine the interrelationship among error patterns, it would present bases for predicting students' errors and misconceptions for meaningful engagement by the teachers and other stakeholders in mathematics education.

Statement of the Problem

In recent years, many research works on mathematics education have focused on learning difficulties of students related to algebra (Wu, 2001; Norton & Irvin, 2007; Egodawatte, 2011; Adeniji & Ibrahim, 2015). Research (Norton & Irvin, 2007), suggested that solutions to the problem of students inability to be successful in algebra are many and interconnected. Many findings have also related students' errors in algebra to fundamental differences between arithmetic and algebra (Wu, 2001; Stacey & Chick, 2004; Norton & Irvin, 2007; Egodawatte, 2011; Adeniji & Ibrahim, 2015). Despite all these researches and their suggestions no attempt has been made by researchers to study the nature of each component of algebraic conceptions in relation to difference mathematical abilities of students in Nigerian secondary schools so as to look deep into the peculiarities of algebraic difficulties of these different groups of students. As a result, this study employs content (Thematic) analysis method to discover major misconceptions of mathematics students of different abilities in basic algebraic expression concepts in senior secondary school. These approaches would not only empower teachers to adopt and implement appropriate strategies in their teachings but also provide insight into how curriculum developers and textbooks writers can professionally engage these misconceptions/errors.

Objectives of the Study

The study has the following objectives:

(i) To identify major misconceptions commonly held by Senior Secondary School Student of different performance ability in algebraic expression.

(ii) To determine the extent to which these major misconceptions predict students' (i) performance ability and (ii) error distributions in algebraic expression concepts.

Research Questions

(i) What are the major misconceptions of high, medium and low ability students in algebraic expression concepts?
(ii) To what extent will significant relationship exist between major misconceptions of high, medium and low ability students and the (i) students' ability level (ii) error distribution in algebraic expression concepts?

Null hypothesis

HO_1: There is no significant relationship between major misconception of high, medium and low ability students and the students' (i) performance ability level (ii) error distributions in algebraic expression concepts.

Methodology

This study adopted descriptive survey research design. According to Fraenkel and Wallen (2006), any data generated to identify important variables and to further generate hypotheses for future research is exploratory. Thus, the data obtained were used for exploratory purposes.

The population for the study consists of all SSS III students in all the seven educational zones of Katsina State of Nigeria. There are 188 senior secondary schools and 30,541 students of SSS III in Katsina state. All SSS III in all the seven educational zones were stratified into the three senatorial districts of the state (North, Central and South) and adequate proportion of students with respect to their ability levels were randomly selected from each. This technique was adopted, as stressed by Fraenkel and Wallen (2006) and Kothari and Garg (2014), because of its strength at increasing the likelihood of representativeness. Therefore, this technique was used for selecting sixty (60) students which consist of 20 students for each ability level.

The study employed two instruments namely: Students Mathematical Ability Test (SMAT) and Students Algebraic Expression Misconception Test (SAEMT). The SMAT was prepared by the researcher to group students into three different mathematical ability levels. The SMAT contained 40 items multiple choice of five options A to E from critical aspects of mathematics curriculum at secondary school level. These aspects are Algebra, Arithmetic Process, Geometry and Probability & Statistics. The SAEMT was prepared to obtain data about students' understanding of algebraic expression concept by including a variety of items. The test contained 5 essay items which further categorized algebraic expression as simplification, equivalence and Building. To ensure the content validity, both SAEMT and SMAT were prepared by consulting the Katsina State Mathematics Teaching Scheme; SSS1-SSS3 (Ministry of Education, 2006) as a basis. The split-half method was used to establish the internal consistence (Reliability) of the two tests after administration to 25 students who did not take part in the study. The reliability coefficient for the whole test (R_f) using the Spearman-Brown prophecy formula was 0.86 and 0.83 for SAEMT and SMAT respectively.

Data Analysis

The data analyses in this study involved two procedures namely: Students grouping and Rubric construction. To group students into different ability levels, procedures from Almeida and Freire in Ferrando et al. (2012) was adopted. Thus, students were grouped into High ability (those with score 75 percentile and above), Medium ability (those with score between 75 and 25 percentile) and Low ability (those with score 25 percentile and below).

For rubric construction, the researcher painstakingly analyzed the students' procedural and conceptual errors in SAEMT in order to infer their misconceptions. The process consisted of the following: identification of errors, description of errors, classification of errors, quantification of errors and making inferences of underlying misconceptions of these errors. This method of analysis used was described by Sambo (2005) and Fraenkel & Wallen (2006) as content (thematic) analysis. Being categorical data, the researcher employed frequency counts, percentages calculations, contingency (Crossbreak) table, Cramer's V (Effect size) to answer the research questions while Chi-square test of independence analysis was used to test the statistical significance of the findings of the study.

Results

Question 1: What are the major misconceptions of high, medium and low ability students in algebraic expression concepts?

Table 1: Distributions of Algebraic Misconceptions and Error Category of High, Medium and Low Ability Students in Expression

Students' ability	Major Misconceptions	Sub – error Category	% Occurrence
High	Invalid/insufficient premise for algebraic manipulation	• Invalid simplification(39.0) • Incomplete simplification (12.2) • Wrong Algorithm(3.7) • Invalid distribution(6.1)	60
	Deficient belief about algebraic structure and restructuring	• Convert algebraic factors & expression to equation(9.6) • Unnecessary balancing(1.2)	10
	Value mind fixed answer	• Obtaining value for algebraic expression(1.2) • Convert algebraic expression to arithmetic(1.2)	2
	Other misconceptions	• Inadequate factorization procedure(6.1) • Misinterpretation error(6.1) • Miscellaneous errors(13.4)	28
Medium	Invalid/insufficient premise for algebraic manipulation	• Invalid simplification(39.0) • Operational deficiency(4.3) • Invalid distribution(5.8) • Incomplete simplification(0.7)	50
	Deficient belief about algebraic structure and restructuring	• Convert algebraic expression to equation(13.0) • Unnecessary balancing(0.7) • Illegal separation of algebraic term(1.4) • Lack knowledge of algebraic structure(2.8)	17

	Value mind fixed answer	• Obtaining value for algebraic expression(1.4) • Convert algebraic expression to arithmetic(2.8) • Substituting arbitrary values for simplification(1.4)	5
	Other Misconceptions	• Misinterpretation error(4.3) • Wrong factors(1.4) • Miscellaneous error(21.7)	28
Low	Invalid/insufficient premise for algebraic manipulations	• Invalid simplification(46.8) • Wrong algorithm(8.5) • Invalid distribution(4.3) • Incomplete simplification(3.2)	63
	Deficient belief about algebraic structure and restructuring	• Convert algebraic expression to equation(4.3) • Illegal separation of algebraic term(1.1) • Lack knowledge of algebraic structure(3.2)	9
	Value mind fixed answer	• Obtaining value for algebraic expression(6.4) • Convert algebraic expression to arithmetic(3.2) • Substituting arbitrary values for simplification(5.3)	15
	Other Misconceptions	• Wrong factorizations(1.1) • Miscellaneous error(11.7) • Reversal error (1.1)	13

Table 1 shows that students of high, medium and low ability build their misconceptions in algebraic expression on the same ground as they all recorded invalid/insufficient premise for algebraic manipulations, deficient belief about algebraic structure and restructuring, value mind fixed answers and other misconceptions with percentage occurrence. Misconception which based on invalid/insufficient premise for algebraic manipulations is a group of students' error categories that describe invalid procedures for expression simplifications, wrong algorithms, invalid distribution for equivalent expression and incomplete simplifications.

Deficient beliefs about algebraic structure and restructuring based misconception described notion of students to illegally change the structure of algebraic expression given in order to simplify, build or show equivalent of expression at will. This misconception has illegal conversion of expression to equation, illegal separation and transformation of algebraic terms, unnecessary balancing and lack knowledge of algebraic structures. The researcher used value mind fixed answers misconception to describe students' error categories which rooted in believing that all mathematical problems must have value answers. This can be seen from the error categories that make up the misconception from obtaining illegal values for algebraic simplification to conversion of algebraic expression to arithmetic and substituting arbitrary values to obtain their answers.

And lastly, other misconceptions are categories of errors that are specific to a certain concept in algebraic expression and errors whose procedures cannot be linked to any known mathematical algorithms but can still be investigated further. These categories include wrong/inadequate factorization technique, misinterpretation errors and miscellaneous errors.

Question 2: To what extent will significant relationship exist between major misconceptions of high, medium and low ability students and the students' (i) ability level (ii) error distributions in algebraic expression concepts?

Table 2: Relationship between students' ability levels, error distributions and common misconceptions in algebraic expression problems

Variables	Row	Column	N	df	χ^2-cal	χ^2-tab	V	Remark
Students ability	4	3	314	6	21.621	18.5476	0.20	*S
Error distributions	4	3	314	6	26.888	22.458	0.21	**S

*S= Significant at p <0.005 **S= Significant at p< 0.001 V= Effect size

Table 2 shows that there is a significant relationship between the students' ability and their common misconceptions in algebraic expression problems χ^2(df= 6, p< 0.005)= 21.621 and the null hypothesis which states that there is no significant relationship between the two variables is rejected. Information provided by the Table 2 also indicates that χ^2 calculated (26.888) at p< 0.001 is greater than χ^2 critical (22.458). Then, the null hypothesis which states that there is no significant relationship is also rejected Thus, we conclude that invalid/insufficient algebraic manipulations, deficient belief about algebraic structure and restructuring, value mind fixed answers and other misconceptions are significantly related to students' ability and error distributions among algebraic expression concepts with small degree of association (V= 0.20, 0.21) respectively.

Discussion

Analysis of students' responses to SAEMT recorded 314 error types. Three and some specific misconceptions termed as "other misconceptions" were inferred as commonly held misconceptions under this concept and they were found to be significantly related to students' abilities and error distributions in algebraic expression with small degree of association (effect size) respectively. Firstly, invalid/insufficient premise for algebraic manipulations based misconception described students belief that is built on unrecognized procedures for algebraic expression simplifications which include over-simplification, incorrect cross multiplication; wrong algorithms, invalid distribution for equivalence expression and incomplete simplifications. This finding supports (Egodawatte, 2011; Adeniji & Ibrahim, 2015).

The reasons given in the literatures for invalid simplification especially oversimplification include conceiving open algebraic expression as 'incomplete' and try to finish them by over-simplification, and due to nature of mathematical notations as processes and objects (Tall & Thomas, 1991; Stacey & McGregor, 1994) or erroneously draw on previous learning from other subjects (Egodawatte, 2011).

Secondly, deficient belief about algebraic structure and restructuring based misconception captured students' misconception that illegally prompted them to change the structure of a given algebraic structure to simplify, build and show its equivalence. One of the error categories under this misconception is illegal conversion of expression to equation which agreed with (Egodawatte, 2011; Adeniji & Ibrahim, 2015). Others are illegal separation/transformation of algebraic terms, unnecessary balancing and inadequate knowledge of algebraic structures.

Thirdly, misconception rooted in value mind fixed answers. This is a students' belief that mathematical problems must have value answers. The error categories that made this conception are obtaining illegal values, substituting arbitrary values for algebraic expression and erroneous conversion of algebraic expression to arithmetic to obtain value answers. This finding is in agreement with (Bodin & Capponi, 1996; Egodawatte, 2011; Adeniji & Ibrahim, 2015). Egodawatte (2011) gave the reason for some of these errors as when students used equal sign to indicate 'the next step is' or, in order words, when the equal was used as a 'step maker' (p.137).

And lastly, some error categories that are only specific to certain concepts in algebraic expression related problems merged together to have a category tagged other misconceptions. These groups of misconceptions consist of misinterpretation/reversal error of word framed questions, wrong factorization techniques and miscellaneous errors. These sorts of errors were also indicated in the literature (Egodawatte, 2011; Adeniji & Ibrahim, 2015). Available literatures lack the information on how major misconceptions of students predict their performance ability in mathematics and error occurrence in algebra concepts.

Conclusion

Based on the procedures and statistical inferences made from this study, it could be concluded that invalid/insufficient premise for algebraic manipulations, deficient belief about algebraic structure and restructuring, value mind fixed answers and some specific misconceptions categories such as misinterpretation/reversal error of word framed questions, wrong factorization techniques and miscellaneous errors are the commonly held misconceptions of high, medium and low ability senior secondary school students in algebraic expression related problems. Furthermore, these misconceptions can predict students' performance ability level in mathematics and their error distributions among algebraic expression concepts to the degree of 20% and 21%. It could also be concluded from the analyses conducted in the study that students of different performance ability level in senior secondary school are likely to hold the same naive belief about algebraic concepts making ability grouping very much unnecessary when teaching algebra concepts.

Recommendations

It becomes imperative to make the following recommendations as a result of the major findings of the study:

(i) Mathematics teachers in secondary school should pay attention to all the misconceptions discovered in the study and design thoughtful instructional methods to handle them in their algebraic classes in respective of the ability level of the students.

(ii) Mathematics teachers especially those in primary schools where the basic concepts of arithmetic start should be exposed to training which will enhance their effective teaching of the concepts. This will play a very prominent role in the mastery of algebraic concepts.

(iii) Curriculum planners and Textbook writers should take into cognizance of all the error categories that constitute students misconceptions when designing curriculum and textbooks to facilitate teachers' algebraic teaching and cancel students naïve belief/notions.
(iv) Researchers in the field of mathematics education should try further to investigate the reasons behind students' errors in concepts such as Evaluation and Comparing algebraic expression that are not investigated in this study.

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